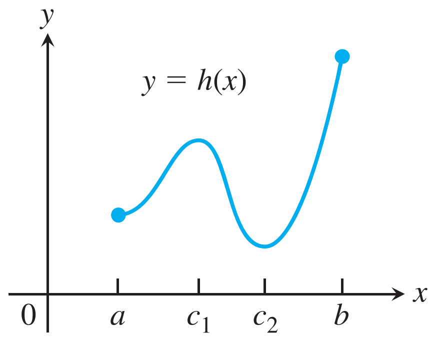
## Section 4.1

**Problem 1.** *Determine from the graph whether the function has any absolute extreme values on* $[a, b]$*. Then explain how your answer is consistent with Theorem 1.*

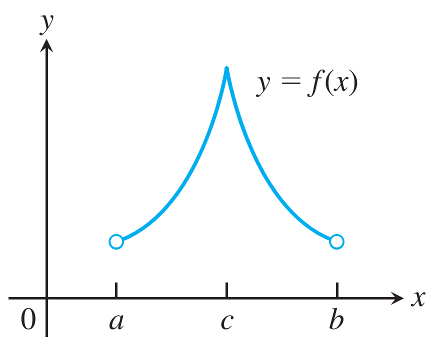



**Solution** There is an $\boxed{\text{absolute max at } x = b}$ and an $\boxed{\text{absolute min at } x = c_2}$

This is consistent with Theorem 1 because $h$ is continuous on the closed interval $[a, b]$.

□

**Problem 3.** *Determine from the graph whether the function has any absolute extreme values on* $[a, b]$*. Then explain how your answer is consistent with Theorem 1.*




**Solution** There is an $\boxed{\text{absolute max at } x = c}$ and $\boxed{\text{no absolute min}}$

Theorem 1 does not apply because the function is not defined at the end points.

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**Problem 15.** *Find the absolute minimum and maximum values of each function on the given interval. Then graph the function. Identify the points on the graph where the absolute extrema occur, and include their coordinates:* $f(x) = \frac{2}{3}x - 5$, $-2 \le x \le 3$

**Solution**

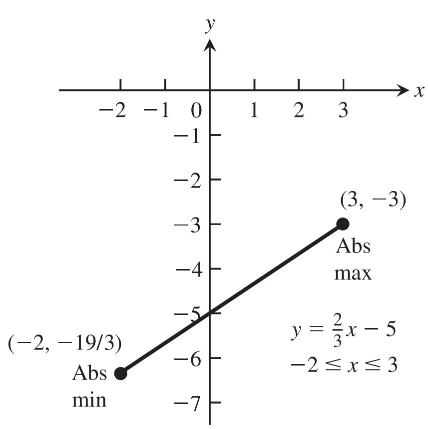
$$f'(x) = \frac{2}{3}$$

So we just have to test the end points:

$$\begin{aligned} f(-2) &= \frac{2}{3}(-2) - 5 \\ &= -\frac{4}{3} - 5 \\ &= -\frac{4}{3} - \frac{15}{3} \\ &= \boxed{-\frac{19}{3} \leftarrow \text{absolute min}} \end{aligned}$$

$$\begin{aligned} f(3) &= \frac{2}{3}(3) - 5 \\ &= 2 - 5 \\ &= \boxed{-3 \leftarrow \text{absolute max}} \end{aligned}$$

Putting this on a graph we have:

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**Problem 17.** *Find the absolute minimum and maximum values of each function on the given interval. Then graph the function. Identify the points on the graph where the absolute extrema occur, and include their coordinates:* $f(x) = x^2 - 1,\ -1 \le x \le 2$

**Solution**

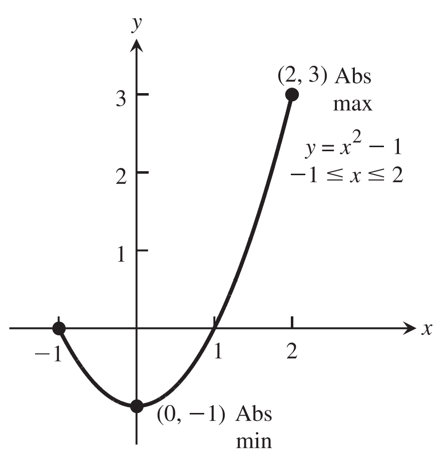
$$f'(x) = 2x \text{ so } f'(x) = 0 \Leftrightarrow x = 0$$

$$\begin{aligned} f(-1) &= (-1)^2 - 1 \\ &= 0 \end{aligned}$$

$$\begin{aligned} f(2) &= 2^2 - 1 \\ &= \boxed{3 \leftarrow \text{absolute max}} \end{aligned}$$

$$\begin{aligned} f(0) &= 0^2 - 1 \\ &= \boxed{-1 \leftarrow \text{absolute min}} \end{aligned}$$

Putting this together we have

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**Problem 19.** *Find the absolute minimum and maximum values of each function on the given interval. Then graph the function. Identify the points on the graph where the absolute extrema occur, and include their coordinates:* $F(x) = -\dfrac{1}{x^2}$, $0.5 \le x \le 2$

**Solution**

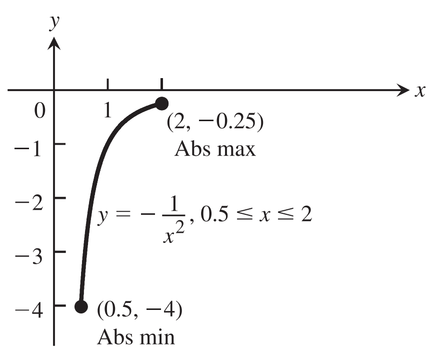
$$F'(x) = \frac{d}{dx}(-x^{-2}) = \frac{2}{x^3}$$

$F'(x)$ is undefined at 0 but so is $F(x)$ so we don't have to check it.

$$\begin{aligned} F(0.5) &= F\left(\frac{1}{2}\right) \\ &= -\frac{1}{(1/2)^2} \\ &= -\frac{1}{1/4} \\ &= \boxed{-4 \leftarrow \text{abs min}} \end{aligned}$$

$$\begin{aligned} F(2) &= -\frac{1}{2^2} \\ &= \boxed{-\frac{1}{4} \leftarrow \text{abs max}} \end{aligned}$$

The graph looks like the following:

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**Problem 25.** *Find the absolute minimum and maximum values of each function on the given interval. Then graph the function. Identify the points on the graph where the absolute extrema occur, and include their coordinates:* $f(\theta) = \sin\theta,\ -\frac{\pi}{2} \le \theta \le \frac{5\pi}{6}$

**Solution**

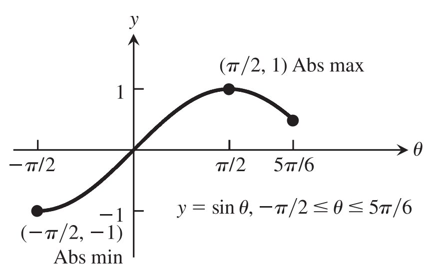
$$f'(\theta) = \cos\theta \text{ so } f'(\theta) = 0 \Rightarrow \theta = -\frac{\pi}{2}, \frac{\pi}{2}$$

$$\begin{aligned} f\left(-\frac{\pi}{2}\right) &= \sin\left(-\frac{\pi}{2}\right) \\ &= \boxed{-1 \leftarrow \text{abs min}} \end{aligned}$$

$$\begin{aligned} f\left(\frac{\pi}{2}\right) &= \sin\left(\frac{\pi}{2}\right) \\ &= \boxed{1 \leftarrow \text{abs max}} \end{aligned}$$

$$\begin{aligned} f\left(\frac{5\pi}{6}\right) &= \sin\left(\frac{5\pi}{6}\right) \\ &= \frac{1}{2} \end{aligned}$$

The graph looks like the following:

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**Problem 35.** *Find the extreme values of the function $y = 2x^2 - 8x + 9$ and where they occur.*

**Solution**

$$\frac{dy}{dx} = 4x - 8$$

So

$$\frac{dy}{dx} = 0 \Leftrightarrow 4x - 8 = 0 \Leftrightarrow x = 2$$

$$y|_{x=2} = 2(4) - 8(2) + 9 = 8 - 16 + 9 = 1$$

The graph is the graph of a upward facing parabola so there is an $\boxed{\text{absolute minimum of 1 at } x = 2}$

□

**Problem 39.** *Find the extreme values of the function $y = \sqrt{x^2 - 1}$ and where they occur.*

**Solution**

$$\frac{dy}{dx} = \frac{1}{\not{2}}(x^2 - 1)^{-1/2}(\not{2}x) = \frac{x}{\sqrt{x^2 - 1}}$$

$$\frac{dy}{dx} = 0 \Leftrightarrow x = 0 \text{ and } \frac{dy}{dx} \text{ is undefined if } x = \pm 1$$

$$y|_{x=\pm 1} = 1 - 1 = 0$$

Thus there are $\boxed{\text{absolute minimums of 0 at } x = \pm 1}$

□