Key 16 17

Do Now:

1. Find the inverse function $g(x)$ for $f(x) = x^3 + 3$.

$y = x^3 + 3$
$x = y^3 + 3$
$y = \sqrt[3]{x-3}$

$\boxed{g(x) = \sqrt[3]{x-3}}$

2. Check your work by verifying $f(x)$ and $g(x)$ are inverses.

$f(g(x)) = (\sqrt[3]{x-3})^3 + 3$
$= x - 3 + 3$
$= x$ ✓

$g(f(x)) = \sqrt[3]{x^3+3-3}$
$= \sqrt[3]{x^3}$
$= x$ ✓

3. If the point (2, 11) is on $f(x)$, what point is on $g(x)$? $\boxed{(11,2)}$

4. Calculate $f'(2)$ and $g'(11)$. What do you notice about the derivatives at the corresponding points?

$f'(x) = 3x^2$ $\quad g'(x) = \frac{1}{3(x-3)^{2/3}}$

$\boxed{f'(2) = 12 \quad g'(11) = \frac{1}{3 \cdot 4} = \frac{1}{12}}$

slopes of tangent lines are reciprocals at corresponding points

Inverse Functions:

A function has an inverse function if and only if it is one-to-one.

If a function is strictly ***monotonic*** on its entire domain, then it is one-to-one and therefore has an inverse.

If the point (a, b) is on the function, then (b, a) is on the inverse function.

The slopes of inverse functions at corresponding points are reciprocals.

Sample Problems:

1. Given the fact that $f(x)$ and $g(x)$ are inverse functions, find $g'(x)$.

$f(g(x)) = x$
$f'(g(x))g'(x) = 1 \Rightarrow \boxed{g'(x) = \frac{1}{f'(g(x))}}$

2. Let $f(x) = \frac{1}{4}x^3 + x - 1$. Let $g(x)$ represent the inverse of $f(x)$. What is the value of $g(x)$ when $x = 3$? What is the value of $g'(x)$ when $x = 3$?

$\frac{f(x)}{(c,3)}$ $\quad \frac{g(x)}{(3,c)} \Rightarrow (3,2)$

$\frac{1}{4}c^3 + c - 1 = 3$
$\frac{1}{4}c^3 + c - 4 = 0$
$c = 2$
$f'(x) = \frac{3}{4}x^2 + 1$
$f'(2) = \frac{3}{4}(2)^2 + 1 = 4 \Rightarrow \boxed{g'(3) = \frac{1}{4}}$

3. Find the derivative of the inverse function of $f(x) = x^3 - 4x^2 + 7x - 1$ at $x = 1$.

$\frac{f(x)}{(c,1)}$ $\quad \frac{f^{-1}(x)}{(1,c)}$

$c^3 - 4c^2 + 7c - 1 = 1$
$c^3 - 4c^2 + 7c - 2 = 0$
$c \approx .349$

$f'(.349) \approx 4.571$

$\boxed{(f^{-1})'(1) \approx .219}$

4. Let $f(x)$ be a differentiable function with values given in the table below. Assume that $f(x)$ has a differentiable inverse function $g(x)$. Complete the table to give as much information as possible for the inverse function.

x	$f(x)$	$f'(x)$
1	-3	4
2	1	5
3	2	6

x	$g(x)$	$g'(x)$
-3	1	$1/4$
1	2	$1/5$
2	3	$1/6$

a. Find an equation of the line tangent to the graph of $g(x)$ at $x=1$. $\boxed{y-2=\frac{1}{5}(x-1)}$

b. Find an equation of the line normal to the graph of $g(x)$ at $x=2$. $\boxed{y-3=-6(x-2)}$

5. Using two different methods, evaluate the derivative of the inverse function of $f(x)=\sqrt{x^2-4}$ at $x=2\sqrt{3}$.

$f(x)=\sqrt{x^2-4}$ D: $[2,\infty)$ R: $[0,\infty)$ Find $f^{-1}(x)$ + derive

$x=\sqrt{y^2-4}$

$x^2+4=y^2$

$f^{-1}(x)=\sqrt{x^2+4}$ D: $[0,\infty)$ R: $[2,\infty)$

$(f^{-1})'(x)=\frac{2x}{2\sqrt{x^2+4}}$ $\Rightarrow (f^{-1})'(2\sqrt{3})=\frac{2\sqrt{3}}{\sqrt{16}}=\frac{2\sqrt{3}}{4}=\frac{\sqrt{3}}{2}$

$f(x)$	$f^{-1}(x)$
$(c, 2\sqrt{3})$	$(2\sqrt{3}, c)$

slopes at corresponding points are reciprocals

$2\sqrt{3}=\sqrt{x^2-4}$

$12=x^2-4$

$x=\pm 4$

$f'(x)=\frac{2x}{2\sqrt{x^2-4}}$

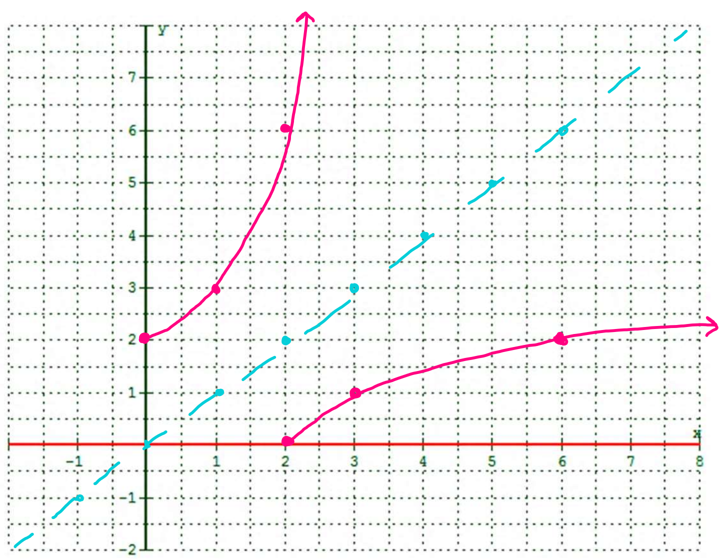
$f'(4)=\frac{4}{2\sqrt{3}}=\frac{2}{\sqrt{3}}$ $\Rightarrow$ $\boxed{(f^{-1})'(2\sqrt{3})=\frac{\sqrt{3}}{2}}$

6. Consider the function $f(x)=x^2+2$ on $[0,\infty)$.

a. Find $f^{-1}(x)$ algebraically. $x=y^2+2$, $y=\sqrt{x-2}$ $\quad f^{-1}(x)=\sqrt{x-2}$

b. Sketch both $f(x)$ and $f^{-1}(x)$ on the axes shown below. What is the relationship between the two graphs?

symmetric to $y=x$



c. Find $f'(x)$ and $(f^{-1})'(x)$.

$f'(x)=2x$

$(f^{-1})'(x)=\frac{1}{2\sqrt{x-2}}$

d. Evaluate $f'(1)$ and $f'(2)$.

$f'(1)=2$ $\quad f'(2)=4$

e. Locate the corresponding points on $f^{-1}(x)$ from part (d). Evaluate $(f^{-1})'(x)$ at these values.

$(f^{-1})'(3)=\frac{1}{2}$ $\quad (f^{-1})'(6)=\frac{1}{4}$

$f(x)$		$f^{-1}(x)$
$(1,3)$	$\rightarrow$	$(3,1)$
$(2,6)$	$\rightarrow$	$(6,2)$

f. What do you observe?

Slopes are reciprocals at corresponding points