Review of inverses:

1. A function f will have an inverse function f^{-1} if and only if f is one-to-one

   - To be one-to-one, a continuous function must be either increasing or decreasing

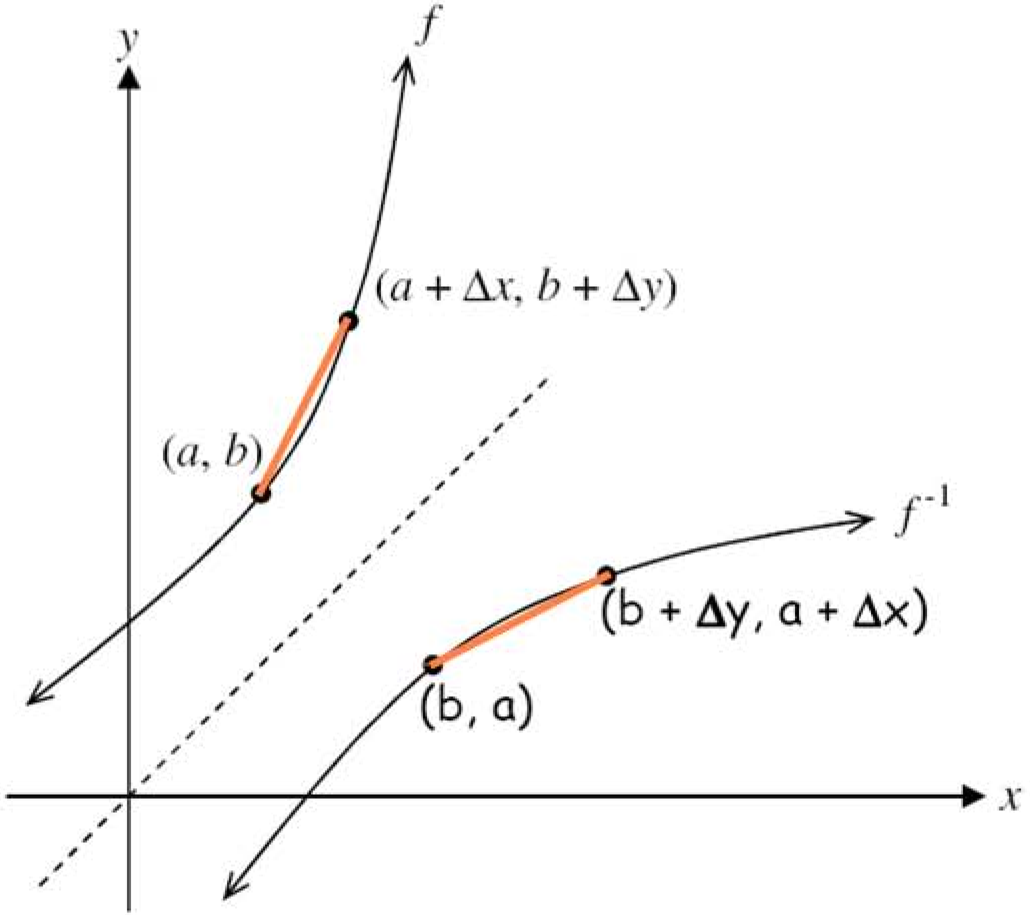
2. The graph of f^{-1} is the reflection of the graph of f over the line $y = x$

3. If f is continuous, then f^{-1} is also continuous.

4. If f is increasing(decreasing), then f^{-1} is also increasing(decreasing).

5. If f is differentiable at $x = c$ and $f'(c) \neq 0$, then f^{-1} is differentiable at $x = f(c)$.

We want a formula for $(f^{-1})'$.



Note: $f(a) = b \rightarrow a = f^{-1}(b)$

$$\left.\begin{aligned} f:\ m_{sec} &= \frac{\Delta y}{\Delta x} \\ f^{-1}:\ m_{sec} &= \frac{\Delta x}{\Delta y} \end{aligned}\right\} \text{Reciprocals}$$

$$\therefore (f^{-1}(b))' = \frac{1}{f'(a)}$$

Inverse functions have reciprocal slopes at reflected points.

$$\therefore (f^{-1}(b))' = \frac{1}{f'(f^{-1}(b))} = \frac{1}{f'(a)}$$

We may substitute x for b, so

$$\frac{d}{dx}(f^{-1}(x)) = \frac{1}{f'(f^{-1}(x))}$$

Analytic Approach

$$y = f'(x) \rightarrow x = f^{-1}(y) \rightarrow 1 = f'(y)y' \rightarrow y' = \frac{1}{f'(y)}$$

$$\therefore \frac{d}{dx}(f^{-1}(x)) = \frac{1}{f'(f^{-1}(x))}$$

(2) Reflect the point, find x for $y = a$ $f(b) = a$	(1) To find $(f^{-1})'(a)$
(3) Find the derivative, evaluate at $x = b$ $f'(b)$	(4) Take the reciprocal of $f'(b)$ $(f^{-1})'(a) = \frac{1}{f'(b)}$

Ex: If $f(4) = 6$ and $f'(4) = \frac{2}{3}$ and $f'(6) = \frac{3}{5}$, find $\left(f^{-1}\right)'(6)$

(2) Reflect the point, find x for $y = 6$ $f(4) = 6$ so $x = 4$	(1) To find $\left(f^{-1}\right)'(6)$
(3) Find the derivative, evaluate at $x = 4$ $f'(4) = \frac{2}{3}$	(4) Take the reciprocal of $f'(4)$ $\left(f^{-1}\right)'(6) = \frac{1}{\frac{2}{3}} = \frac{3}{2}$

Ex: If $f(x)=\frac{x^3}{32}+x-5$, find $\left(f^{-1}\right)'(1)$.

(1) To find

$$\left(f^{-1}\right)'(1)$$

(2) Reflect the point, find x for $y = 1$ (use calc)

$$\frac{x^3}{32}+x-5=1 \text{ so } x=4$$

(3) Find the derivative, evaluate at $x = 4$

$$f'(x)=\frac{3x^2}{32}+1$$

$$f'(4)=\frac{5}{2}$$

(4) Take the reciprocal of $f'(4)$

$$\left(f^{-1}\right)'(1)=\frac{2}{5}$$

Ex: If $g(x) = x^5 + 2x - 1$, find $\frac{d}{dx} g^{-1}(2)$.

(2) Reflect the point, find x for $y = 2$ (use calc) $x^5 + 2x - 1 = 2$ so $x = 1$	(1) To find $\left(g^{-1}\right)'(2)$
(3) Find the derivative, evaluate at $x = 1$ $g'(x) = 5x^4 + 2$ $g'(1) = 7$	(4) Take the reciprocal of $g'(1)$ $\left(g^{-1}\right)'(2) = \frac{1}{7}$

Ex. Selected values for the functions $f(x)$ and $f'(x)$ are shown in the table below. Let $g(x) = f^{-1}(x)$.

x	1	2	3	4
$f(x)$	2	4	1	3
$f'(x)$	1	3	4	2

a) Find $g'(3)$

(2) Reflect the point, find x for $y = 3$

$f(x) = 3$ so $x = 4$

(1) To find

$(g)'(3)$

(3) Find the derivative, evaluate at $x = 4$

$f'(4) = 2$

(4) Take the reciprocal of $f'(4)$

$(g)'(3) = \frac{1}{2}$

b) Find $g'(4)$

(2) Reflect the point, find x for $y = 4$ $f(x) = 4$ so $x = 2$	(1) To find $(g)'(4)$
(3) Find the derivative, evaluate at $x = 2$ $f'(2) = 3$	(4) Take the reciprocal of $f'(2)$ $(g)'(4) = \frac{1}{3}$