Worksheet —Implicit Differentiation

Show all work. No calculator unless otherwise stated.

1. Find $\frac{dy}{dx}$

(a) $x^3 - 3x^2y + 4xy^2 = 12$

$\frac{d}{dx}[x^3 - 3x^2y + 4xy^2] = \frac{d}{dx}[12]$

$3x^2 - 6xy - 3x^2\frac{dy}{dx} + 4y^2 + 4x(2y\frac{dy}{dx}) = 0$

$\frac{dy}{dx}[-3x^2 + 8xy] = 6xy - 3x^2 - 4y^2$

$\frac{dy}{dx} = \frac{6xy - 3x^2 - 4y^2}{8xy - 3x^2}$

or $\frac{dy}{dx} = \frac{3x^2 + 4y^2 - 6xy}{3x^2 - 8xy}$ *multiply by $\frac{-1}{-1}$

(b) $\sqrt{xy} = x + 3y$

$\frac{d}{dx}[(xy)^{1/2}] = \frac{d}{dx}[x + 3y]$

$\frac{1}{2}(xy)^{-1/2}(1\cdot y + x\cdot\frac{dy}{dx}) = 1 + 3\frac{dy}{dx}$

$\frac{1}{2}x^{-1/2}y^{-1/2}y + \frac{1}{2}x^{-1/2}x y^{-1/2}\frac{dy}{dx} = 1 + 3\frac{dy}{dx}$

$\frac{\sqrt{y}}{2\sqrt{x}} + \frac{\sqrt{x}}{2\sqrt{y}}\frac{dy}{dx} = 1 + 3\frac{dy}{dx}$

$\frac{\sqrt{y}}{2\sqrt{x}} - 1 = \frac{dy}{dx}\left[3 - \frac{\sqrt{x}}{2\sqrt{y}}\right]$

$\frac{dy}{dx} = \frac{\frac{\sqrt{y}}{2\sqrt{x}} - 1}{3 - \frac{\sqrt{x}}{2\sqrt{y}}}\left(\frac{2\sqrt{x}\sqrt{y}}{2\sqrt{x}\sqrt{y}}\right)$

$\frac{dy}{dx} = \frac{y - 2\sqrt{xy}}{6\sqrt{xy} - x}$

or $\frac{dy}{dx} = \frac{2\sqrt{xy} - y}{x - 6\sqrt{xy}}$ *multiply by $\frac{-1}{-1}$

(c) $4\sin 2y\cos x = 2$

$\frac{d}{dx}[4\sin(2y)\cos x] = \frac{d}{dx}[2]$

$4\cos(2y)\cdot 2\frac{dy}{dx}\cdot\cos x + 4\sin(2y)(-\sin x) = 0$

$8\cos(2y)\cos(x)\frac{dy}{dx} = 4\sin(2y)\sin x$

$\frac{dy}{dx} = \frac{\sin(2y)\sin x}{2\cos(2y)\cos x}$

or $\frac{dy}{dx} = \frac{1}{2}\tan(2y)\tan x$

(d) $(y^2 + 2\sec y)^2 = 4(x+1)^2$

$\frac{d}{dx}[(y^2 + 2\sec y)^2] = \frac{d}{dx}[4(x+1)^2]$

$2(y^2 + 2\sec y)^1\cdot(2y\frac{dy}{dx} + 2\sec y\tan y\cdot\frac{dy}{dx}) = 8(x+1)^1(1)$

$\frac{dy}{dx}(2y + 2\sec y\tan y) = \frac{8(x+1)}{2(y^2 + 2\sec y)}$

$\frac{dy}{dx} = \frac{4(x+1)}{(y^2 + 2\sec y)(2)(y + \sec y\tan y)}$

$\frac{dy}{dx} = \frac{2(x+1)}{(y^2 + 2\sec y)(y + \sec y\tan y)}$

(e) $x = y\sec\left(\frac{5}{y}\right)$

$\frac{d}{dx}[x] = \frac{d}{dx}\left[y\sec\left(\frac{5}{y}\right)\right]$

$1 = \frac{dy}{dx}\sec\left(\frac{5}{y}\right) + y\cdot\sec\left(\frac{5}{y}\right)\tan\left(\frac{5}{y}\right)\cdot\left(\frac{-5}{y^2}\cdot\frac{dy}{dx}\right)$

$1 = \frac{dy}{dx}\left[\sec\left(\frac{5}{y}\right) - \frac{5}{y}\sec\left(\frac{5}{y}\right)\tan\left(\frac{5}{y}\right)\right]$

$\frac{dy}{dx} = \frac{1}{\sec\left(\frac{5}{y}\right) - \frac{5}{y}\sec\left(\frac{5}{y}\right)\tan\left(\frac{5}{y}\right)}$

2. Find $\frac{dy}{dx}$ at the indicated point, then find the equation of both the tangent and normal lines.

(a) $y^2 = \frac{x^2-4}{x^2+4}$ at $(2,0)$

$\frac{d}{dx}\left[y^2\right] = \frac{d}{dx}\left[\frac{x^2-4}{x^2+4}\right]$

$2y\frac{dy}{dx} = \frac{(x^2+4)(2x)-(x^2-4)(2x)}{(x^2+4)^2}$

$\frac{dy}{dx} = \frac{2x^3+8x-2x^3+8x}{2y(x^2+4)^2}$

$\boxed{\frac{dy}{dx} = \frac{8x}{y(x^2+4)^2}}$

$\left.\frac{dy}{dx}\right|_{(2,0)} = \frac{16}{0} = \infty$

So the graph has a Vertical tangent line of $x=2$ at $(2,0)$.

The Normal line at $(2,0)$ is the horizontal line $y=0$

(b) $(x+y)^3 = x^3+y^3$ at $(-1,1)$

$\frac{d}{dx}\left[(x+y)^3\right] = \frac{d}{dx}\left[x^3+y^3\right]$

$3(x+y)^2\left(1+\frac{dy}{dx}\right) = 3x^2+3y^2\frac{dy}{dx}$

$3(x+y)^2 + 3\frac{dy}{dx}(x+y)^2 = 3x^2+3y^2\frac{dy}{dx}$

$\frac{dy}{dx}\left[3(x+y)^2-3y^2\right] = 3x^2-3(x+y)^2$

$\boxed{\frac{dy}{dx} = \frac{x^2-(x+y)^2}{(x+y)^2-y^2}}$

$\left.\frac{dy}{dx}\right|_{(-1,1)} = \frac{1-0^2}{0^2-1} = -1 = m$

Tangent Line: $\boxed{y = 1-1(x+1)}$

Normal line: $\boxed{y = 1+1(x+1)}$

3. Find $\frac{d^2y}{dx^2}$ in terms of x and y.

(a) $x^2+y^2 = 36$

$\frac{d}{dx}\left[x^2+y^2\right] = \frac{d}{dx}[36]$

$2x+2y\frac{dy}{dx} = 0$

$\boxed{\frac{dy}{dx} = \frac{-x}{y}}$

$\frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d}{dx}\left[\frac{-x}{y}\right]$

$\frac{d^2y}{dx^2} = \frac{(y)(-1)-(-x)\frac{dy}{dx}}{y^2}$

$\frac{d^2y}{dx^2} = \frac{-y+x\left(-\frac{x}{y}\right)}{y^2}\left(\frac{y}{y}\right)$

$\boxed{\frac{dy}{dx} = \frac{-y^2-x^2}{y^3}}$

or $\frac{dy}{dx} = -\frac{x^2+y^2}{y^3}$

(b) $1-xy = x-y$

*Solve for y 1st!

$y-xy = x-1$

$y(1-x) = x-1$

$y = \frac{x-1}{-(x-1)}$

$y = -1,\ x\neq 1$

$\frac{dy}{dx} = 0,\ x\neq 1$

$\boxed{\frac{d^2y}{dx^2} = 0,\ x\neq 1}$

* simplify early & often

(c) $\sqrt[3]{x^2}+\sqrt[3]{y^2} = 1$

*Solve for y 1st!

$y^{2/3} = 1-x^{2/3}$

$y = (1-x^{2/3})^{3/2}$

$\frac{dy}{dx} = \frac{3}{2}(1-x^{2/3})^{1/2}\cdot\left(-\frac{2}{3}x^{-1/3}\right)$

$\frac{dy}{dx} = \left(-x^{-1/3}\right)\left(1-x^{2/3}\right)^{1/2}$

$\frac{d^2y}{dx^2} = \left(\frac{1}{3}x^{-4/3}\right)\left(1-x^{2/3}\right)^{1/2} + \left(-x^{-1/3}\right)\left(\frac{1}{2}(1-x^{2/3})^{-1/2}\cdot\left(-\frac{2}{3}x^{-1/3}\right)\right)$

$\frac{d^2y}{dx^2} = \frac{1}{3}x^{-4/3}(1-x^{2/3})^{1/2} + \frac{1}{3}x^{-2/3}(1-x^{2/3})^{-1/2}$

$\frac{d^2y}{dx^2} = \frac{1}{3}x^{-4/3}(1-x^{2/3})^{-1/2}\left[(1-x^{2/3})+x^{2/3}\right]$

factor out least powers!

$\boxed{\frac{d^2y}{dx^2} = \frac{1}{3\sqrt[3]{x^4}\sqrt{1-\sqrt[3]{x^2}}}}$

4. Determine the point(s) at which the graph of $y^4 = y^2 - x^2$ has either a horizontal or vertical tangent. Be sure to label which is which, if either exist.

$\frac{d}{dx}[y^4] = \frac{d}{dx}[y^2 - x^2]$

$4y^3 \cdot \frac{dy}{dx} = 2y\frac{dy}{dx} - 2x$

$\frac{dy}{dx}[4y^3 - 2y] = -2x$

$\frac{dy}{dx} = \frac{-2x}{4y^3 - 2y}$

$\frac{dy}{dx} = \frac{2x}{2y - 4y^3}$

Horz. tangents when $\frac{dy}{dx} = \frac{0}{\neq 0}$

$2x = 0$

$x = 0$

when $x=0$:

$y^4 = y^2 - 0^2$

$y^4 - y^2 = 0$

$y^2(y^2 - 1) = 0$

$y = 0, y = -1, y = 1$

pts: $(0,0), (0,-1), (0,1)$

Vert. tangents when $\frac{dy}{dx} = \frac{\neq 0}{0}$

$2y - 4y^3 = 0$

$2y(1 - 2y^2) = 0$

$y = 0, \quad y = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2}$

when $y=0$: $0^4 = 0^2 - x^2$, $x = 0$

when $y = \pm\frac{\sqrt{2}}{2}$:

$\left(\pm\frac{\sqrt{2}}{2}\right)^4 = \left(\pm\frac{\sqrt{2}}{2}\right)^2 - x^2$

$\frac{4}{16} = \frac{2}{4} - x^2$

$\frac{1}{4} - \frac{1}{2} = -x^2$

$x = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$

pts: $(0,0), \left(\frac{1}{2}, \pm\frac{\sqrt{2}}{2}\right), \left(-\frac{1}{2}, \pm\frac{\sqrt{2}}{2}\right)$

5. Find the two points where the curve $x^2 + xy + y^2 = 7$ crosses the x-axis, and show that the tangents to the curve at these points are parallel. What is the common slope of these tangents?

when curve crosses x-axis, $y=0$:

$x^2 + 0 + 0 = 7$

$x = -\sqrt{7}$ or $x = \sqrt{7}$

when $x = \sqrt{7}$:

$7 + \sqrt{7}y + y^2 = 7$

$y(\sqrt{7} + y) = 0$

$y = 0$ or $y = -\sqrt{7}$

pts: $(\sqrt{7}, 0), (\sqrt{7}, -\sqrt{7})$

when $x = -\sqrt{7}$:

$7 - \sqrt{7}y + y^2 = 7$

$y(y - \sqrt{7}) = 0$

$y = 0, y = \sqrt{7}$

pts: $(-\sqrt{7}, 0), (-\sqrt{7}, \sqrt{7})$

So the TWO x-intercepts of the curve are $(-\sqrt{7}, 0)$ and $(\sqrt{7}, 0)$ (as shown in the graph at right)

parallel tangent lines have the same slope.

$\frac{d}{dx}[x^2 + xy + y^2] = \frac{d}{dx}[7]$

$2x + (1)(y) + (x)\left(\frac{dy}{dx}\right) + (2y)\left(\frac{dy}{dx}\right) = 0$

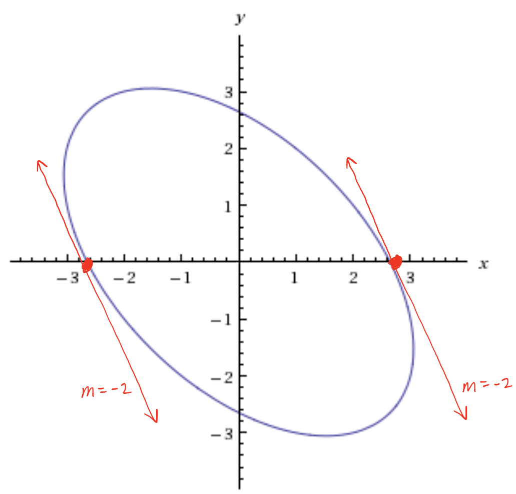
$\frac{dy}{dx}(x + 2y) = -2x - y$

$\frac{dy}{dx} = -\frac{2x + y}{x + 2y}$

$\left.\frac{dy}{dx}\right|_{(-\sqrt{7}, 0)} = -\frac{-2\sqrt{7} + 0}{-\sqrt{7} + 2(0)} = -2$

$\left.\frac{dy}{dx}\right|_{(\sqrt{7}, 0)} = -\frac{2\sqrt{7} + 0}{\sqrt{7} + 2(0)} = -2$

So the common slope is -2 at the x-intercepts.



6. Find the equations of the normal lines to the curve $xy + 2x - y = 0$ that are parallel to the line $2x + y = 0$

$2x + y = 0$

$y = -2x$ ← slope $= -2$

if the normal lines are to be parallel the given line, their slopes must equal -2, so the tangent lines at these points must be $\frac{1}{2}$

$\frac{d}{dx}[xy + 2x - y] = \frac{d}{dx}[0]$

$(1)(y) + (x)\left(\frac{dy}{dx}\right) + 2 - \frac{dy}{dx} = 0$

$\frac{dy}{dx}(x - 1) = -y - 2$

$\frac{dy}{dx} = \frac{-y-2}{x-1}$

$\frac{dy}{dx} = \frac{y+2}{1-x}$

So $\frac{dy}{dx} = \frac{1}{2}$

$\frac{y+2}{1-x} = \frac{1}{2}$

$2y + 4 = 1 - x$

$x = 1 - 4 - 2y$

$x = -3 - 2y$

When $x = -3 - 2y$:

$(-3-2y)y + 2(-3-2y) - y = 0$

$-3y - 2y^2 - 6 - 4y - y = 0$

$-2y^2 - 8y - 6 = 0$

$-2(y^2 + 4y + 3) = 0$

$-2(y+1)(y+3) = 0$

$y = -1$ or $y = -3$

when $y = -1$:

$-x + 2x + 1 = 0$

$x = -1$

pt: $(-1, -1)$

Normal line eq: $m = -2$, pt $(-1,-1)$

$y = -1 - 2(x+1)$

when $y = -3$:

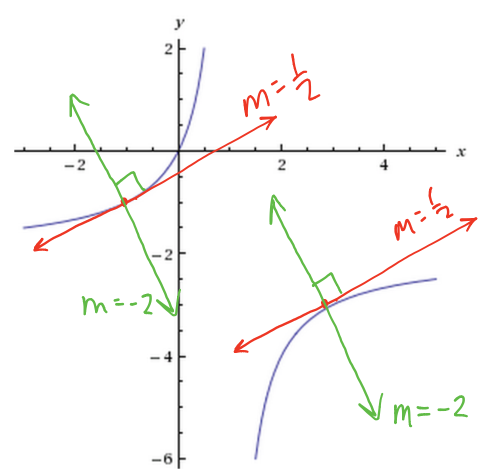
$-3x + 2x + 3 = 0$

$x = 3$

pt: $(3, -3)$

Normal Line Eq: $m = -2$, pt $(3, -3)$

$y = -3 - 2(x - 3)$



7. If $y^2 + \cos xy - 4x = 5$, find $\dfrac{dx}{dy}$, yes, that's $\dfrac{dx}{dy}$.

$\frac{d}{dy}\left[y^2+\cos xy-4x\right]=\frac{d}{dy}[5]$

$2y\frac{dy}{dy}+(-\sin xy)\left(\frac{dx}{dy}y+x\frac{dy}{dy}\right)-4\frac{dx}{dy}=0$

$2y-\sin(xy)\left(y\frac{dx}{dy}+x\right)-4\frac{dx}{dy}=0$

$2y-y\frac{dx}{dy}\sin(xy)-x\sin(xy)-4\frac{dx}{dy}=0$

$\frac{dx}{dy}\left(-y\sin(xy)-4\right)=x\sin(xy)-2y$

$\frac{dx}{dy}=\frac{x\sin(xy)-2y}{-y\sin(xy)-4}$

$\boxed{\frac{dx}{dy}=\frac{2y-x\sin(xy)}{y\sin(xy)+4}}$

8. The slope of the tangent is -1 at the point $(0,1)$ on $x^3 - 6xy - ky^3 = a$, where k and a are constants. The values of the constants a and k are what?

At $(0,1)$:
$0^3-6(0)(1)-k(1^3)=a$
$\boxed{-k=a}$

$\frac{d}{dx}\left[x^3-6xy-ky^3\right]=\frac{d}{dx}[a]$

$3x^2-6y-6x\frac{dy}{dx}-3ky^2\frac{dy}{dx}=0$

$\frac{dy}{dx}\left(-6x-3ky^2\right)=6y-3x^2$

$\frac{dy}{dx}=\frac{6y-3x^2}{-6x-3ky^2}$

$\boxed{\frac{dy}{dx}=\frac{3x^2-6y}{6x+3ky^2}}$

$\left.\frac{dy}{dx}\right|_{(0,1)}=-1=\frac{3(0^2)-6(1)}{6(0)+3k(1^2)}$

$-1=\frac{-6}{3k}$

$3k=6$
$\boxed{k=2}$

So if $-k=a$
$a=-k$
When $k=2$, $a=-2$

Multiple Choice

B 9. Find y' when $xy + 5x + 2x^2 = 4$.

(A) $y' = \dfrac{5+2x-y}{x}$ (B) $y' = -\dfrac{y+5+4x}{x}$ (C) $y' = -(y+5+4x)$ (D) $y' = \dfrac{y+5+2x}{x}$

(E) $y' = -\dfrac{y+5+2x}{x}$ (F) $y' = \dfrac{y+5+4x}{x}$

this equation is easily solvable for y, BUT the answer choices all have y in them, so we will differentiate implicitly.

$\frac{d}{dx}\left[xy+5x+2x^2\right]=\frac{d}{dx}[4]$

$(1)(y)+(x)\left(\frac{dy}{dx}\right)+5+4x=0$

$x\frac{dy}{dx}=-y-5-4x$

$\boxed{\frac{dy}{dx}=-\frac{y+5+4x}{x}}$ B

A ___ 10. Find $\frac{dy}{dx}$ when $\frac{3}{\sqrt{x}} + \frac{2}{\sqrt{y}} = 4$

(A) $\frac{dy}{dx} = -\frac{3}{2}\left(\frac{y}{x}\right)^{3/2}$ (B) $\frac{dy}{dx} = \frac{3}{2}(xy)^{1/2}$ (C) $\frac{dy}{dx} = -\frac{2}{3}\left(\frac{x}{y}\right)^{3/2}$ (D) $\frac{dy}{dx} = \frac{2}{3}\left(\frac{x}{y}\right)^{3/2}$ (E) $\frac{dy}{dx} = \frac{3}{2}\left(\frac{y}{x}\right)^{3/2}$ (F) $\frac{dy}{dx} = \frac{2}{3}(xy)^{1/2}$

Same as #9. Don't solve for y since answer choices have y in them.

$\frac{d}{dx}\left[3x^{-1/2} + 2y^{-1/2}\right] = \frac{d}{dx}[4]$

$-\frac{3}{2}x^{-3/2} - y^{-3/2} \cdot \frac{dy}{dx} = 0$

$-\frac{3}{2}x^{-3/2} = y^{-3/2}\frac{dy}{dx}$

$\frac{dy}{dx} = \left(-\frac{3}{2}\right)\left(\frac{x^{-3/2}}{y^{-3/2}}\right)$

$\frac{dy}{dx} = \left(-\frac{3}{2}\right)\left(\frac{x}{y}\right)^{-3/2}$

$\frac{dy}{dx} = \left(-\frac{3}{2}\right)\left(\frac{y}{x}\right)^{3/2}$ [A]

D ___ 11. Find the equation of the tangent line to the graph of $y^2 - xy - 12 = 0$ at the point $(1,4)$.

(A) $3y = 2x + 10$ (B) $3y + 2x = 10$ (C) $y = 4x$ (D) $7y = 4x + 24$ (E) $7y + 4x = 24$

$\frac{d}{dx}\left[y^2 - xy - 12\right] = \frac{d}{dx}[0]$

$2y\frac{dy}{dx} - (1)(y) - (x)\left(\frac{dy}{dx}\right) = 0$

$\frac{dy}{dx}(2y - x) = y$

$\frac{dy}{dx} = \frac{y}{2y - x}$

$\left.\frac{dy}{dx}\right|_{(1,4)} = \frac{4}{8-1} = \frac{4}{7} = m$

eq: $7\left[y = 4 + \frac{4}{7}(x-1)\right]7$

$7y = 28 + 4(x-1)$

$7y = 28 + 4x - 4$

$7y = 4x + 24$ [D]

E ___ 12. The slope of the tangent line to the graph of $x^3 - 2y^3 + xy = 0$ at the point $(1,1)$ is

(A) $-\frac{4}{5}$ (B) $\frac{3}{2}$ (C) $-\frac{5}{4}$ (D) $\frac{5}{4}$ (E) $\frac{4}{5}$ (F) $-\frac{2}{3}$

$\frac{d}{dx}\left[x^3 - 2y^3 + xy\right] = \frac{d}{dx}[0]$

$3x^2 - 6y^2\frac{dy}{dx} + (1)(y) + (x)\left(\frac{dy}{dx}\right) = 0$

At (1,1): $3 - 6\frac{dy}{dx} + 1 + \frac{dy}{dx} = 0$

$-5\frac{dy}{dx} = -4$

$\left.\frac{dy}{dx}\right|_{(1,1)} = \frac{4}{5}$ [E]

B 13. The points P and Q on the graph of $y^2 - xy + 8 = 0$ have the same x-coordinate, $x = 6$. The point of intersection of the tangents to the graph at P and Q is

(A) $\left(\frac{8}{3}, \frac{16}{3}\right)$ (B) $\left(\frac{16}{3}, \frac{8}{3}\right)$ (C) $\left(\frac{16}{3}, \frac{16}{3}\right)$ (D) $\left(\frac{8}{3}, \frac{8}{3}\right)$ (E) $\left(\frac{8}{3}, \frac{2}{3}\right)$

when $x=6$: $y^2-6y+8=0$
$(y-4)(y-2)=0$
$y=4, y=2$
pts: $(6,4), (6,2)$

$\frac{d}{dx}\left[y^2-xy+8\right]=\frac{d}{dx}[0]$
$2y\frac{dy}{dx}-(1)(y)-(x)\left(\frac{dy}{dx}\right)=0$
$\frac{dy}{dx}(2y-x)=y$
$\boxed{\frac{dy}{dx}=\frac{y}{2y-x}}$

$\left.\frac{dy}{dx}\right|_{(6,4)}=\frac{4}{8-6}=2$
tangent line @ $(6,4)$: $y=4+2(x-6)$

$\left.\frac{dy}{dx}\right|_{(6,2)}=\frac{2}{4-6}=-1$
tangent line @ $(6,2)$: $y=2-(x-6)$

The tangent Lines intersect...
$4+2(x-6)=2-(x-6)$
$4+2x-12=2-x+6$
$3x=16$
$\boxed{x=\frac{16}{3}}$
when $x=\frac{16}{3}$:
$y=4+2\left(\frac{16}{3}-6\right)$
$y=4+\frac{32}{3}-12$
$y=-8+\frac{32}{3}$
$y=\frac{32-24}{3}=\frac{8}{3}$
pt of intersection: $\left(\frac{16}{3},\frac{8}{3}\right)$ B

D 14. Determine $\frac{d^2y}{dx^2}$ when $4x^2 + 3y^2 = 4$

(A) $\frac{d^2y}{dx^2} = \frac{16}{9y^2}$ (B) $\frac{d^2y}{dx^2} = -\frac{16}{9y^2}$ (C) $\frac{d^2y}{dx^2} = -\frac{4}{9y^3}$ (D) $\frac{d^2y}{dx^2} = -\frac{16}{9y^3}$ (E) $\frac{d^2y}{dx^2} = \frac{16}{9y^3}$

$\frac{d}{dx}\left[4x^2+3y^2\right]=\frac{d}{dx}[4]$
$8x+6y\frac{dy}{dx}=0$
$\frac{dy}{dx}=\frac{-8x}{6y}$
$\boxed{\frac{dy}{dx}=\frac{-4x}{3y}}$

$\frac{d}{dx}\left[\frac{dy}{dx}\right]=\frac{d}{dx}\left[\frac{-4x}{3y}\right]$
$\frac{d^2y}{dx^2}=\frac{(3y)(-4)-(-4x)\left(3\frac{dy}{dx}\right)}{(3y)^2}$
$\frac{d^2y}{dx^2}=\frac{-12y+12x\frac{dy}{dx}}{9y^2}$
$\frac{d^2y}{dx^2}=\frac{-12y+12x\left(\frac{-4x}{3y}\right)}{9y^2}\left(\frac{y}{y}\right)$
$\frac{d^2y}{dx^2}=\frac{-12y^2-16x^2}{9y^3}$

*Notice all answer choices are in terms of y!
$4x^2=(4-3y^2)$
So $\frac{d^2y}{dx^2}=\frac{-12y^2-4(4x^2)}{9y^3}$
$\frac{d^2y}{dx^2}=\frac{-12y^2-4(4-3y^2)}{9y^3}$
$\frac{d^2y}{dx^2}=\frac{-12y^2-16+12y^2}{9y^3}$
$\boxed{\frac{d^2y}{dx^2}=\frac{-16}{9y^3}}$ D

F ____ 15. When an object is placed at a distance p from a convex lens having focal length 5 cm, the image will be at a distance q cm from the lens, while $\frac{1}{5}=\frac{1}{p}+\frac{1}{q}$. Find the rate of change of p with respect to q.

(A) $\frac{dp}{dq}=\frac{5}{q-5}$ (B) $\frac{dp}{dq}=\frac{25}{q-5}$ (C) $\frac{dp}{dq}=\frac{25}{(q-5)^2}$ (D) $\frac{dp}{dq}=-\frac{25}{q-5}$

(E) $\frac{dp}{dq}=-\frac{5}{(q-5)^2}$ (F) $\frac{dp}{dq}=-\frac{25}{(q-5)^2}$

Want $\frac{dp}{dq}$. q is independent variable, p is dependent variable. ** Solve the equation for p as a function of q. ** All answer choices are a function of q.

$\frac{1}{5}=\frac{1}{p}+\frac{1}{q}$

$\frac{1}{5}=\frac{q+p}{qp}$

$qp=5q+5p$ (cross multiply)

$qp-5p=5q$

$p(q-5)=5q$

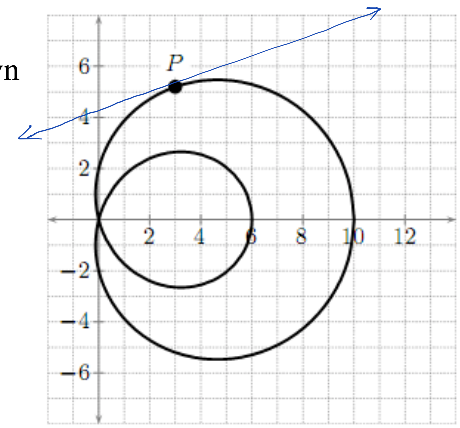
$p=\frac{5q}{q-5}$

So $\frac{dp}{dq}=\frac{(q-5)(5)-(5q)(1)}{(q-5)^2}$

$\frac{dp}{dq}=\frac{5q-25-5q}{(q-5)^2}$

$\frac{dp}{dq}=\frac{-25}{(q-5)^2}$ F

D ____ 16. The graph of the equation $\left(x^2+y^2-8x\right)^2=4\left(x^2+y^2\right)$ is shown at right. Find the equation of the tangent line to the graph at the point $\left(3,3\sqrt{3}\right)$.



(A) $y=\frac{1}{\sqrt{3}}\left(\frac{5}{3}x-12\right)$ (B) $y=\frac{1}{\sqrt{3}}\left(\frac{5}{3}x+12\right)$ (C) $y=\frac{1}{\sqrt{3}}\left(\frac{3}{5}x+\frac{6\sqrt{3}}{5}\right)$

(D) $y=\frac{1}{\sqrt{3}}\left(\frac{3}{5}x+\frac{36}{5}\right)$ (E) $y=\frac{1}{\sqrt{3}}\left(\frac{3}{5}x+\frac{6}{5}\right)$

$\frac{d}{dx}\left[(x^2+y^2-8x)^2\right]=\frac{d}{dx}\left[4(x^2+y^2)\right]$

$2(x^2+y^2-8x)^1\cdot\left(2x+2y\frac{dy}{dx}-8\right)=4\left(2x+2y\frac{dy}{dx}\right)$

At $(3,3\sqrt{3})$:

$2(9+27-24)\left(6+6\sqrt{3}\frac{dy}{dx}-8\right)=4\left(6+6\sqrt{3}\frac{dy}{dx}\right)$

$6\,24\left(-2+6\sqrt{3}\frac{dy}{dx}\right)=4\left(6+6\sqrt{3}\frac{dy}{dx}\right)$

$-12+36\sqrt{3}\frac{dy}{dx}=6+6\sqrt{3}\frac{dy}{dx}$

$30\sqrt{3}\frac{dy}{dx}=18$

$\frac{dy}{dx}=\frac{18}{30\sqrt{3}}$

$\frac{dy}{dx}=\frac{3}{5\sqrt{3}}=m$

eq: $y=3\sqrt{3}+\frac{3}{5\sqrt{3}}(x-3)$

$y=3\sqrt{3}+\frac{3}{5\sqrt{3}}x-\frac{9}{5\sqrt{3}}$

$y=\frac{3}{5\sqrt{3}}x+\frac{3\sqrt{3}}{1}\left(\frac{5\sqrt{3}}{5\sqrt{3}}\right)-\frac{9}{5\sqrt{3}}$

$y=\frac{3}{5\sqrt{3}}x+\frac{45-9}{5\sqrt{3}}$

$y=\frac{3}{5\sqrt{3}}x+\frac{36}{5\sqrt{3}}$

$y=\frac{1}{\sqrt{3}}\left(\frac{3}{5}x+\frac{36}{5}\right)$ D

Kuta Software - Infinite Calculus Implicit Differentiation

For each problem, use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y.

1) $2x^3 = 2y^2 + 5$

$\frac{dy}{dx} = \frac{3x^2}{2y}$

2) $3x^2 + 3y^2 = 2$

$\frac{dy}{dx} = -\frac{x}{y}$

3) $5y^2 = 2x^3 - 5y$

$\frac{dy}{dx} = \frac{6x^2}{10y + 5}$

4) $4x^2 = 2y^3 + 4y$

$\frac{dy}{dx} = \frac{4x}{3y^2 + 2}$

5) $5x^3 = -3xy + 2$

$\frac{dy}{dx} = \frac{-y - 5x^2}{x}$

6) $1 = 3x + 2x^2y^2$

$\frac{dy}{dx} = \frac{-3 - 4xy^2}{4x^2y}$

7) $3x^2y^2 = 4x^2 - 4xy$

$\frac{dy}{dx} = \frac{4x - 2y - 3xy^2}{3x^2y + 2x}$

8) $5x^3 + xy^2 = 5x^3y^3$

$\frac{dy}{dx} = \frac{15x^2y^3 - 15x^2 - y^2}{2xy - 15x^3y^2}$

9) $2x^3 = (3xy + 1)^2$

$\frac{dy}{dx} = \frac{-3y^2x - y + x^2}{3x^2y + x}$

10) $x^2 = (4x^2y^3 + 1)^2$

$\frac{dy}{dx} = \frac{-32y^6x^2 - 8y^3 + 1}{48x^3y^5 + 12xy^2}$

11) $\sin 2x^2y^3 = 3x^3 + 1$

$\frac{dy}{dx} = \frac{9x - 4y^3\cos 2x^2y^3}{6xy^2\cos 2x^2y^3}$

12) $3x^2 + 3 = \ln 5xy^2$

$\frac{dy}{dx} = \frac{6yx^2 - y}{2x}$

For each problem, use implicit differentiation to find $\frac{d^2y}{dx^2}$ in terms of x and y.

13) $4y^2 + 2 = 3x^2$

$\frac{d^2y}{dx^2} = \frac{12y^2 - 9x^2}{16y^3}$

14) $5 = 4x^2 + 5y^2$

$\frac{d^2y}{dx^2} = \frac{-20y^2 - 16x^2}{25y^3}$

Implicit differentiation worksheet

Solutions:

1. $\frac{1}{2y}$
2. $-\frac{3x}{y}$
3. $\frac{3-2x}{8y-10}$
4. $-\frac{2x\sin(x)+x^2\cos(x)}{2y\cos(y)-y^2\sin(y)}$
5. $-\frac{y}{x}$
6. $(1-y^2\sec^2 x)/2y\tan(x)$
7. $\frac{8x+3y^2-12xy}{3y^2+6x^2-6xy}$
8. $-\frac{2y^{\frac{5}{2}}}{1+4xy^{\frac{3}{2}}}$
9. $\frac{-2(y+2)}{x}$
10. $(2\sqrt{\tan(y)}/(\sec^2 y)$
11. $(\sqrt{\tan(y)}/(y\sec^2 y)$