F Business and Economic Applications

- **Understand basic business terms and formulas, determine marginal revenues, costs and profits, find demand functions, and solve business and economics optimization problems.**

Business and Economics Applications

Previously, you learned that one of the most common ways to measure change is with respect to time. In this section, you will study some important rates of change in economics that are not measured with respect to time. For example, economists refer to **marginal profit, marginal revenue,** and **marginal cost** as the rates of change of the profit, revenue, and cost with respect to the number of units produced or sold.

SUMMARY OF BUSINESS TERMS AND FORMULAS

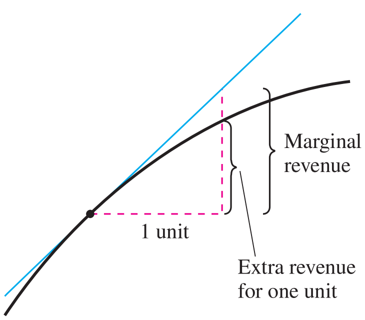
Basic Terms	**Basic Formulas**
x is the number of units produced (or sold).	
p is the price per unit.	
R is the total revenue from selling x units.	$R = xp$
C is the total cost of producing x units.	
$\overline{C}$ is the average cost per unit.	$\overline{C} = \dfrac{C}{x}$
P is the total profit from selling x units.	$P = R - C$
The **break-even point** is the number of units for which $R = C$.	

Marginals

$\dfrac{dR}{dx}$ = marginal revenue ≈ *extra* revenue from selling one additional unit

$\dfrac{dC}{dx}$ = marginal cost ≈ *extra* cost of producing one additional unit

$\dfrac{dP}{dx}$ = marginal profit ≈ *extra* profit from selling one additional unit



A revenue function
Figure F.1

In this summary, note that marginals can be used to approximate the *extra* revenue, cost, or profit associated with selling or producing one additional unit. This is illustrated graphically for marginal revenue in Figure F.1.

EXAMPLE 1 Finding the Marginal Profit

A manufacturer determines that the profit P (in dollars) derived from selling x units of an item is given by

$$P = 0.0002x^3 + 10x.$$

a. Find the marginal profit for a production level of 50 units.

b. Compare this with the actual gain in profit obtained by increasing production from 50 to 51 units.

Solution

a. Because the profit is $P = 0.0002x^3 + 10x$, the marginal profit is given by the derivative

$$\frac{dP}{dx} = 0.0006x^2 + 10.$$

When $x = 50$, the marginal profit is

$$\frac{dP}{dx} = (0.0006)(50)^2 + 10 \qquad \text{Marginal profit for } x = 50$$
$$= \$11.50.$$

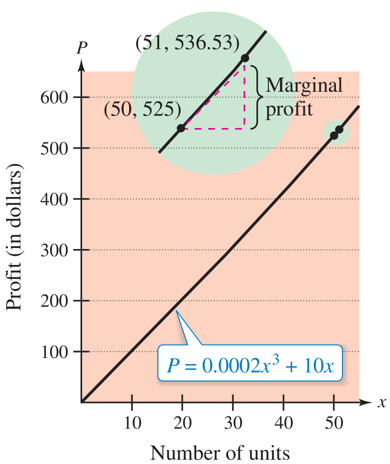
b. For $x = 50$ and 51, the actual profits are

$$P = (0.0002)(50)^3 + 10(50)$$
$$= 25 + 50$$
$$= \$525.00$$
$$P = (0.0002)(51)^3 + 10(51)$$
$$= 26.53 + 510$$
$$= \$536.53.$$

So, the additional profit obtained by increasing the production level from 50 to 51 units is

$$536.53 - 525.00 = \$11.53. \qquad \text{Extra profit for one unit}$$

Note that the actual profit increase of \$11.53 (when x increases from 50 to 51 units) can be approximated by the marginal profit of \$11.50 per unit (when $x = 50$), as shown in Figure F.2.

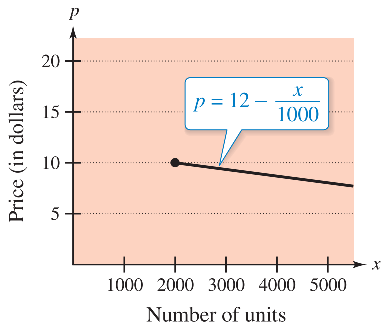


Marginal profit is the extra profit from selling one additional unit.
Figure F.2

The profit function in Example 1 is unusual in that the profit continues to increase as long as the number of units sold increases. In practice, it is more common to encounter situations in which sales can be increased only by lowering the price per item. Such reductions in price ultimately cause the profit to decline.

The number of units x that consumers are willing to purchase at a given price per unit p is given by the **demand function**

$$p = f(x). \qquad \text{Demand function}$$



A demand function p
Figure F.3

EXAMPLE 2 Finding a Demand Function

A business sells 2000 items per month at a price of \$10 each. It is estimated that monthly sales will increase by 250 items for each \$0.25 reduction in price. Find the demand function corresponding to this estimate.

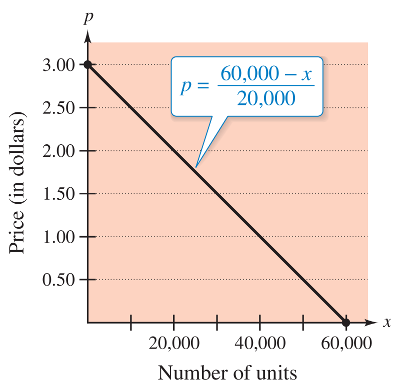
Solution From the estimate, x increases 250 units each time p drops \$0.25 from the original cost of \$10. This is described by the equation

$$x = 2000 + 250\left(\frac{10 - p}{0.25}\right)$$
$$= 12{,}000 - 1000p$$

or

$$p = 12 - \frac{x}{1000}, \quad x \geq 2000. \qquad \text{Demand function}$$

The graph of the demand function is shown in Figure F.3.



As the price decreases, more hamburgers are sold.
Figure F.4

EXAMPLE 3 Finding the Marginal Revenue

A fast-food restaurant has determined that the monthly demand for its hamburgers is

$$p = \frac{60{,}000 - x}{20{,}000}.$$

Find the increase in revenue per hamburger (marginal revenue) for monthly sales of 20,000 hamburgers. (See Figure F.4.)

Solution Because the total revenue is given by $R = xp$, you have

$$R = xp = x\left(\frac{60{,}000 - x}{20{,}000}\right) = \frac{1}{20{,}000}(60{,}000x - x^2).$$

By differentiating, you can find the marginal revenue to be

$$\frac{dR}{dx} = \frac{1}{20{,}000}(60{,}000 - 2x).$$

When $x = 20{,}000$, the marginal revenue is

$$\frac{dR}{dx} = \frac{1}{20{,}000}[60{,}000 - 2(20{,}000)]$$
$$= \frac{20{,}000}{20{,}000}$$
$$= \$1 \text{ per unit.}$$

The demand function in Example 3 is typical in that a high demand corresponds to a low price, as shown in Figure F.4.

EXAMPLE 4 Finding the Marginal Profit

For the fast-food restaurant in Example 3, the cost C (in dollars) of producing x hamburgers is

$$C = 5000 + 0.56x, \quad 0 \le x \le 50{,}000.$$

Find the total profit and the marginal profit for 20,000, 24,400, and 30,000 units.

Solution Because $P = R - C$, you can use the revenue function in Example 3 to obtain

$$P = \frac{1}{20{,}000}(60{,}000x - x^2) - 5000 - 0.56x$$

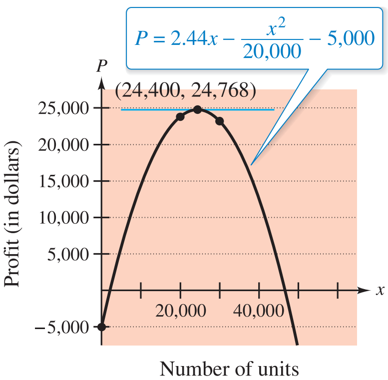
$$= 2.44x - \frac{x^2}{20{,}000} - 5000.$$

So, the marginal profit is

$$\frac{dP}{dx} = 2.44 - \frac{x}{10{,}000}.$$

The table shows the total profit and the marginal profit for each of the three indicated demands. Figure F.5 shows the graph of the profit function.

Demand	20,000	24,400	30,000
Profit	\$23,800	\$24,768	\$23,200
Marginal profit	\$0.44	\$0.00	−\$0.56



The maximum profit corresponds to the point where the marginal profit is 0. When more than 24,400 hamburgers are sold, the marginal profit is negative—increasing production beyond this point will *reduce* rather than increase profit.

Figure F.5

EXAMPLE 5 Finding the Maximum Profit

In marketing an item, a business has discovered that the demand for the item is represented by

$$p = \frac{50}{\sqrt{x}}. \qquad \text{Demand function}$$

The cost C (in dollars) of producing x items is given by $C = 0.5x + 500$. Find the price per unit that yields a maximum profit.

Solution From the cost function, you obtain

$$P = R - C = xp - (0.5x + 500). \qquad \text{Primary equation}$$

Substituting for p (from the demand function) produces

$$P = x\left(\frac{50}{\sqrt{x}}\right) - (0.5x + 500) = 50\sqrt{x} - 0.5x - 500.$$

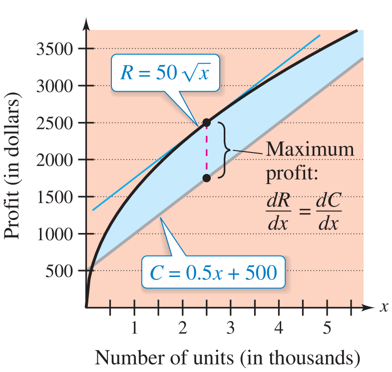
Setting the marginal profit equal to 0

$$\frac{dP}{dx} = \frac{25}{\sqrt{x}} - 0.5 = 0$$

yields $x = 2500$. From this, you can conclude that the maximum profit occurs when the price is

$$p = \frac{50}{\sqrt{2500}} = \frac{50}{50} = \$1.00.$$

See Figure F.6.



Maximum profit occurs when $\frac{dR}{dx} = \frac{dC}{dx}$.

Figure F.6

To find the maximum profit in Example 5, the profit function, $P = R - C$, was differentiated and set equal to 0. From the equation

$$\frac{dP}{dx} = \frac{dR}{dx} - \frac{dC}{dx} = 0$$

it follows that the maximum profit occurs when the marginal revenue is equal to the marginal cost, as shown in Figure F.6.

EXAMPLE 6 Minimizing the Average Cost

A company estimates that the cost C (in dollars) of producing x units of a product is given by $C = 800 + 0.04x + 0.0002x^2$. Find the production level that minimizes the average cost per unit.

Solution Substituting from the equation for C produces

$$\overline{C} = \frac{C}{x} = \frac{800 + 0.04x + 0.0002x^2}{x} = \frac{800}{x} + 0.04 + 0.0002x.$$

Next, find $d\overline{C}/dx$.

$$\frac{d\overline{C}}{dx} = -\frac{800}{x^2} + 0.0002$$

Then, set $d\overline{C}/dx$ equal to 0 and solve for x.

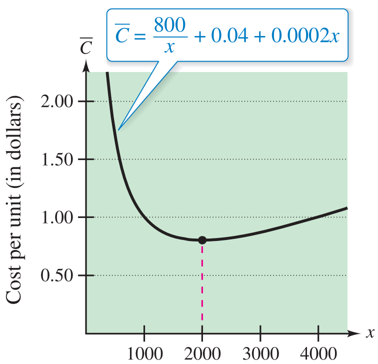
$$-\frac{800}{x^2} + 0.0002 = 0$$

$$0.0002 = \frac{800}{x^2}$$

$$x^2 = 4{,}000{,}000$$

$$x = 2000 \text{ units}$$

See Figure F.7.



Minimum average cost occurs when $\frac{d\overline{C}}{dx} = 0$.

Figure F.7

F Exercises

1. Think About It The figure shows the cost C of producing x units of a product.

(a) What is $C(0)$ called?

(b) Sketch a graph of the marginal cost function.

(c) Does the marginal cost function have an extremum? If so, describe what it means in economic terms.

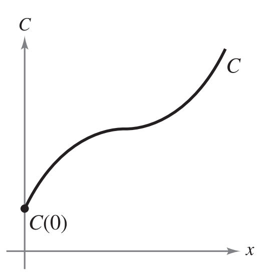


Figure for 1

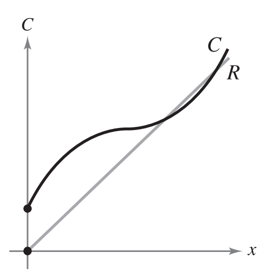


Figure for 2

2. Think About It The figure shows the cost C and revenue R for producing and selling x units of a product.

(a) Sketch a graph of the marginal revenue function.

(b) Sketch a graph of the profit function. Approximate the value of x for which profit is maximum.

Maximum Revenue **In Exercises 3–6, find the number of units x that produces a maximum revenue R.**

3. $R = 900x - 0.1x^2$

4. $R = 600x^2 - 0.02x^3$

5. $R = \dfrac{1{,}000{,}000x}{0.02x^2 + 1800}$

6. $R = 30x^{2/3} - 2x$