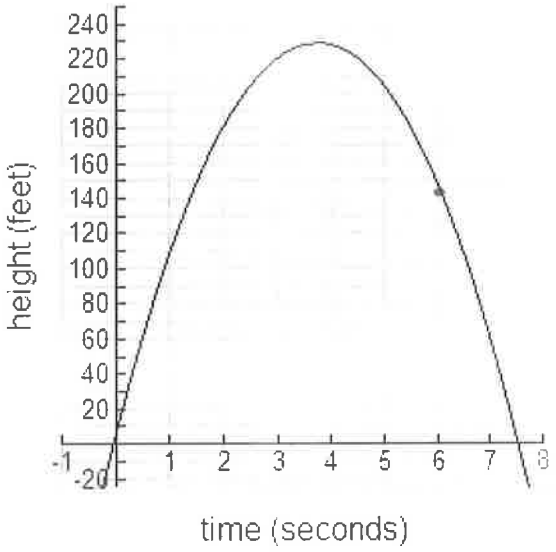
AP Calculus AB

Name KEY

3-4 Velocity and Other Rates of Change

- I can use derivatives to analyze straight line motion and solve other problems involving rates of change.

The below graph models the height over time of an object that was launched straight up into the air. The equation for the graph is $h(t) = 4 + 120t - 16t^2$.



1. Find $h(6)$. Label your answer with units.

   What does your answer mean in the context of the problem?

   $h(6) =$ 148 feet

   Which means ... the object is 148 feet in the air at 6 seconds

2. Find $h'(6)$. Label your answer with units.

   What does your answer mean in the context of the problem?

   $h'(t) =$ $120 - 32t$

   $h'(6) =$ -72 ft/sec

   Which means ... The velocity of the object is -72 ft/sec (72 ft/sec in a negative direction)

3. Find $h''(6)$. Label your answer with units. What does your answer mean in the context of the problem?

   $h''(6) =$ -32 $\frac{\text{ft/sec}}{\text{sec}}$

   Which means ... The acceleration of the object is -32 $\frac{\text{ft/sec}}{\text{sec}}$ → The velocity is changing at a rate of -32 $\frac{\text{ft/sec}}{\text{sec}}$

Notes:

$s(t)$ = position (feet)

$V(t) = s'(t)$ = velocity (ft/sec) → instantaneous rate of change

$a(t) = V'(t) = s''(t)$ = acceleration → rate of change of a rate of change → how fast the velocity is changing

Needed Vocabulary:

Displacement - the amount of movement of an object measured in a particular direction

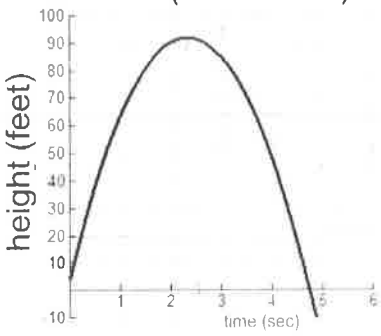
Average Velocity - $\frac{\text{displacement}}{\text{travel time}}$; graphically represented by a secant line; note that when we simply say "velocity", the assumption is we are referring to instantaneous velocity.

Speed – absolute value of velocity

Jerk – derivative of acceleration; third derivative of position

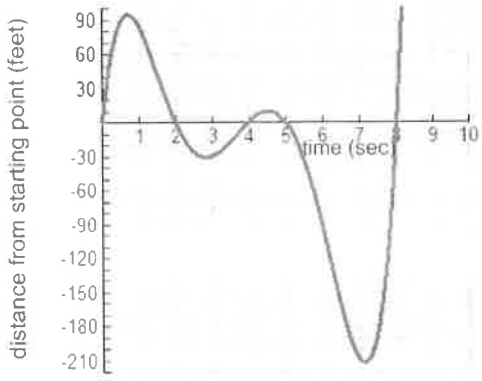
Based on what we learned in the last lesson, when will a secant line overestimate velocity? When will it underestimate velocity? overestimate when graph is concave up; underestimate when concave down

Below is the height of a projectile (*in feet*) over time (*in seconds*) launched into the air



When is the projectile *speeding up*? From its maximum to the ground [note velocity is constantly decreasing]

When is the projectile *slowing down*? from its start to its maximum [speeding up when velocity is moving away from zero]



When is the particle *speeding up*?
When increasing and concave up
When decreasing and concave down (velocity & accel have same sign)

When is the particle *slowing down*?
When increasing and concave down
When decreasing and concave up (velocity & accel have opposite signs)

If an object is traveling at a velocity of –200 ft/sec , what does the negative sign imply?

Moving in a negative direction (usually left/down)

If the object's velocity gets "more negative", what does that mean about the object?

getting further from zero

If the object's velocity gets "less negative", what does that mean about the object?

getting closer to zero

Example 1 – *Calculator active but explain your answers!*

A dynamite blast propels a heavy rock straight up into the air with a launch velocity of 160 ft/sec (about 109 mph). It reaches a height of $s = 160t - 16t^2$ ft after t seconds.

(a) How high does the rock go?

(b) What is the velocity and speed of the rock when it is 256 feet above the ground on the way up? On the way down?

(c) What is the acceleration of the rock at any time t during its flight (after the blast)?

(d) What is the acceleration of the rock at $t = 6$ seconds?

(e) When does the rock hit the ground?

(a) Find maximum of s or when $\frac{ds}{dt} = 0$

$x = 5$

(b) $256 = 160t - 16t^2$

$16t^2 - 160t + 256 = 0$

$t = 2$ or $t = 8$

$s' = 160 - 32t$

$s'(2) = 160 - 32(2) = 96$ ft/sec

$s'(8) = 160 - 32(8) = -96$ ft/sec

The rock is 256 feet above the ground on the way up at $t = 2$. Its velocity is 96 ft/sec

The rock is 256 feet above the ground on the way down at $t = 8$. Its velocity is -96 ft/sec

The speed at both times is 96 ft/sec

(c) $s''(t) =$ $-32 \frac{\text{ft/sec}}{\text{sec}}$

(d) $0 = 160t - 16t^2$

$t = 10$ seconds

Example 2 – Particle Motion – *NO CALCULATOR*

A particle is moving along the horizontal axis in such a way that its position at time t is given by the following function:

$$s(t) = t^3 - 6t^2 + 9t, \quad 0 \le t \le 5$$

(a) Determine a formula for the velocity of the particle.
(b) Determine a formula for the acceleration of the particle.
(c) For what values of t is the particle at rest? *Hint: What is its velocity when it's at rest?*
(d) When is the particle moving to the right? To the left? *Hint: NUMBER LINE ANALYSIS!*
(e) What is the velocity of the particle when the acceleration is zero?
(f) At $t = 4$ seconds is the particle speeding up or slowing down? *Think about the relationship between velocity and acceleration.*
(g) What is the displacement of the particle at $t = 3$
(h) When $t = 3$, what is the total distance traveled by the particle. *Refer back to your NLA analysis!*

(a) $v(t) = s'(t) = 3t^2 - 12t + 9$

(b) $a(t) = s''(t) = 6t - 12$

(c) When the particle is at rest, velocity $= 0$

$3t^2 - 12t + 9 = 0 \rightarrow 3(t^2 - 4t + 3) = 0 \rightarrow 3(t-3)(t-1) = 0$

The particle is at rest at times $t = 1$ and $t = 3$

(d) The particle is moving to the right when velocity is positive

The particle is moving to the right $[0, 1)$ and $(3, 5]$. The particle is moving left $(1, 3)$

(e) Acceleration is zero when $t = 2$; $s'(2) = 3(-1)(1) = \boxed{-3}$

(f) $v(4) = 3(1)(3) = 9$; $a(4) = 6(4) - 12 = 12$; since both acceleration and velocity are positive, the particle is speeding up

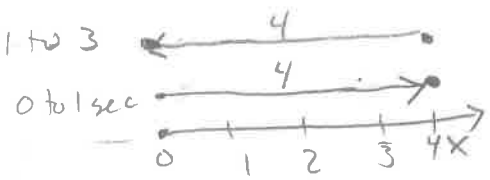
(g) $s(0) = 0$

$s(3) = 3^3 - 6(3^2) + 9(3) = 27 - 54 + 27 = 0$ > Displacement is zero; The particle is back where it started

(h) $s(0) = 0$

$s(1) = 1 - 6 + 9 = 4$

$s(3) = 0$



total distance traveled from at $t = 3$ is 8 units

STOP, DROP, AND AP!

Answer the following AP multiple choice questions:

NO CALCULATOR

A particle moves along the x-axis so that at time $t \geq 0$ its position is given by $x(t) = 2t^3 - 21t^2 + 72t - 53$. At what time t is the particle at rest?

(A) $t = 1$ only

(B) $t = 3$ only

(C) $t = \frac{7}{2}$ only

(D) $t = 3$ and $t = \frac{7}{2}$

(E) $t = 3$ and $t = 4$

$x'(t) = 6t^2 - 42t + 72$

$= 6(t^2 - 7t + 12)$

$= 6(t-4)(t-3)$

$t = 3$ and $t = 4$

$6 \overline{)72}$ 12, 6, 12

CALCULATOR

A particle moves along the x-axis so that at any time $t \geq 0$, its velocity is given by $v(t) = 3 + 4.1\cos(0.9t)$. What is the acceleration of the particle at time $t = 4$?

(A) -2.016 (B) -0.677 (C) 1.633 (D) 1.814 (E) 2.978

$v'(4) = 1.633$ (done on calc)

The velocity, in ft/sec, of a particle moving along the x-axis is given by the function $v(t) = e^t + te^t$. What is the average velocity of the particle from time $t = 0$ to time $t = 3$?

(A) 20.086 ft/sec

(B) 26.447 ft/sec

(C) 32.809 ft/sec

(D) 40.671 ft/sec

(E) 79.342 ft/sec

Let f be the function with derivative given by $f'(x) = \sin(x^2 + 1)$. How many relative extrema does f have on the interval $2 < x < 4$?

(A) One (B) Two (C) Three (D) Four (E) Five

Relative extrema occur when $f'(x) = 0$

4 x-intercepts $2 < x < 4$

Below is an actual free response question from the 2013 AP Calculus Exam. This question was calculator active. Complete the below problem, then see Mr. Sheppard for an answer key. Skip part (b) as we have not done any integrals yet (however note how material from much later in the school year will be used in a problem that we are learning at this point in the school year).

A particle moves along a straight line. For $0 \le t \le 5$, the velocity of the particle is given by $v(t) = -2 + (t^2 + 3t)^{6/5} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$.

(a) Find all values of t in the interval $2 \le t \le 4$ for which the speed of the particle is 2.

(b) Write an expression involving an integral that gives the position $s(t)$. Use this expression to find the position of the particle at time $t = 5$.

(c) Find all times t in the interval $0 \le t \le 5$ at which the particle changes direction. Justify your answer.

(d) Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.

(a) speed is $|v(t)|$

$t = 3.128$ and $t = 3.473$

(b) We'll drop this knowledge later in the course

(c) The partical changes direction when velocity changes sign.

The velocity changes from positive to negative when $t = 3.318$ and from negative to positive when $t = 0.536$ [both $t = 3.318$ and $t = 0.536$ or zeros of $v(t)$]

(d) $v(4) = -11.476 < 0$

$v'(4) = -22.296 < 0$

Since both velocity and acceleration are negative, the speed of the particle is <u>increasing</u>