

Particle Motion Problems
AP Calculus

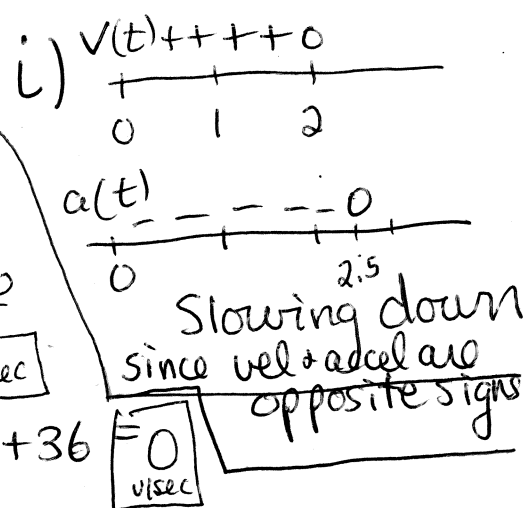
Name: **Answers**

1) A particle moves along the x-axis such that its position at any time t where $0 \leq t \leq 5$ is given by the function $x(t) = 2t^3 - 15t^2 + 36t - 22$

- a) determine the velocity and acceleration functions
- b) what is the particle's average velocity from $t = 2$ to $t = 4$
- c) what is the particle's instantaneous velocity at $t = 3$
- d) when is the particle at rest
- e) when does the particle move to the right
- f) what is the total distance traveled by the particle
- g) what is the particle's maximum velocity
- h) is the particle moving towards or away from the origin at $t = 1$
- i) is the particle speeding up or slowing down at $t = 1$

h) $x(1) = 1$
Particle starts at 1
& moves right
away from origin

a) $v(t) = x'(t) = 6t^2 - 30t + 36$
 $a(t) = v'(t) = x''(t) = 12t - 30$

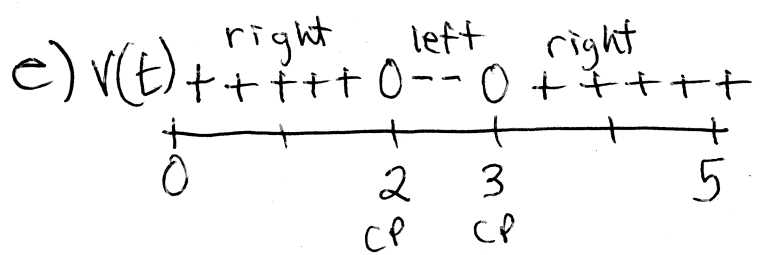


b) average velocity = $\frac{x(4) - x(2)}{4 - 2} = \frac{-10 - 6}{2} = -2 \text{ u/sec}$

c) instan vel. is $x'(3) = 6(3)^2 - 30(3) + 36 = 0 \text{ u/sec}$

d) $v(t) = 0 \quad 6t^2 - 30t + 36 = 0$
 $6(t^2 - 5t + 6) = 0$
 $6(t - 2)(t - 3) = 0$
 $t = 2, t = 3$

Particle is at rest
at $t = 2, t = 3$



moves right $(0, 2) \cup (3, 5)$
moves left $(2, 3)$
 $2 < t < 3$

f) total distance = $|x(0) - x(2)| + |x(2) - x(3)| + |x(3) - x(5)|$
 $= |-22 - 6| + |6 - 5| + |15 - 33| = 57$

g) max velocity is at $t = 0 + t = 5$
 $v(0) = 6(0)^2 - 30(0) + 36 = 36$
 $v(5) = 6(5)^2 - 30(5) + 36 = 36$
since $t = 0 + 5$ are abs max of $v(t)$

Try this without a calculator.

2) A particle starts at time $t = 0$ and moves on a number line so that its position at time t seconds is given by $x(t) = (t-2)^3(t-6)$. Show all work that leads to your answers or justify your answer in words.

- Write the particle's velocity function
- When does the particle stop?
- Does the particle change direction at all its stops?
- What is the particle's displacement from $t = [1,6]$?
- What is the total distance the particle traveled from $t = [1,6]$?
- Set up an equation that could calculate a time when the particle's instantaneous velocity is equal to its average velocity over the interval $[1,6]$. Which theorem does this illustrate?

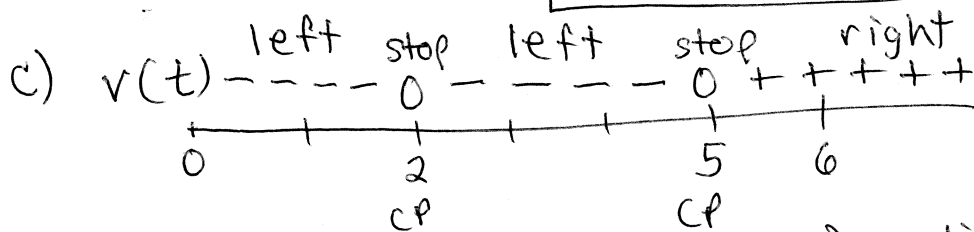
a) $v(t) = x'(t) = 3(t-2)^2(t-6) + (t-2)^3 \cdot (1)$

b) Particle stops when $v(t) = 0$

$$\begin{aligned} (t-2)^2(3(t-6) + (t-2)) &= 0 \\ (t-2)^2(3t-18+t-2) &= 0 \\ (t-2)^2(4t-20) &= 0 \end{aligned}$$

← factor out GCF of $(t-2)$ to solve

$$\boxed{t=2, t=5}$$



No, the particle does not change direction at all its stops! It moves left from 0 sec to 2 sec, then stops, then continues left, then stops, then it turns around & moves right.

d) displacement = $x(6) - x(1) = ((6-2)^3(6-6)) - ((1-2)^3(1-6))$
 $= 0 - 5$
 $= \boxed{-5}$

e) total distance =

$$\begin{aligned} |x(0) - x(5)| + |x(5) - x(6)| &= |48 - -27| + |-27 - 0| \\ &= 75 + 27 = \boxed{102 \text{ units}} \end{aligned}$$

f) ave vel from $[1,6] = \frac{x(6) - x(1)}{6-1} = \frac{-5}{5} = -1$
 $-1 = \frac{3(t-2)^2(t-6) + (t-2)^3}{6-1}$ Mean Value Theorem