Differentiation Rules

General Formulas

Assume u and v are differentiable functions of x.

Constant: $\frac{d}{dx}(c) = 0$

Sum: $\frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}$

Difference: $\frac{d}{dx}(u - v) = \frac{du}{dx} - \frac{dv}{dx}$

Constant Multiple: $\frac{d}{dx}(cu) = c\frac{du}{dx}$

Product: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + \frac{du}{dx}v$

Quotient: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Power: $\frac{d}{dx}x^n = nx^{n-1}$

Chain Rule: $\frac{d}{dx}(f(g(x)) = f'(g(x)) \cdot g'(x)$

Trigonometric Functions

$\frac{d}{dx}(\sin x) = \cos x$ $\qquad$ $\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\tan x) = \sec^2 x$ $\qquad$ $\frac{d}{dx}(\sec x) = \sec x \tan x$

$\frac{d}{dx}(\cot x) = -\csc^2 x$ $\qquad$ $\frac{d}{dx}(\csc x) = -\csc x \cot x$

Exponential and Logarithmic Functions

$\frac{d}{dx}e^x = e^x$ $\qquad$ $\frac{d}{dx}\ln x = \frac{1}{x}$

$\frac{d}{dx}a^x = a^x \ln a$ $\qquad$ $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$

Inverse Trigonometric Functions

$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1 - x^2}}$ $\qquad$ $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1 - x^2}}$

$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1 + x^2}$ $\qquad$ $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2 - 1}}$

$\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1 + x^2}$ $\qquad$ $\frac{d}{dx}(\csc^{-1} x) = -\frac{1}{|x|\sqrt{x^2 - 1}}$

Hyperbolic Functions

$\frac{d}{dx}(\sinh x) = \cosh x$ $\qquad$ $\frac{d}{dx}(\cosh x) = \sinh x$

$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$ $\qquad$ $\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$

$\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$ $\qquad$ $\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$

Inverse Hyperbolic Functions

$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1 + x^2}}$ $\qquad$ $\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2 - 1}}$

$\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1 - x^2}$ $\qquad$ $\frac{d}{dx}(\operatorname{sech}^{-1} x) = -\frac{1}{x\sqrt{1 - x^2}}$

$\frac{d}{dx}(\coth^{-1} x) = \frac{1}{1 - x^2}$ $\qquad$ $\frac{d}{dx}(\operatorname{csch}^{-1} x) = -\frac{1}{|x|\sqrt{1 + x^2}}$

Parametric Equations

If $x = f(t)$ and $y = g(t)$ are differentiable, then

$$y' = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} \quad \text{and} \quad \frac{d^2y}{dx^2} = \frac{dy'/dt}{dx/dt}.$$