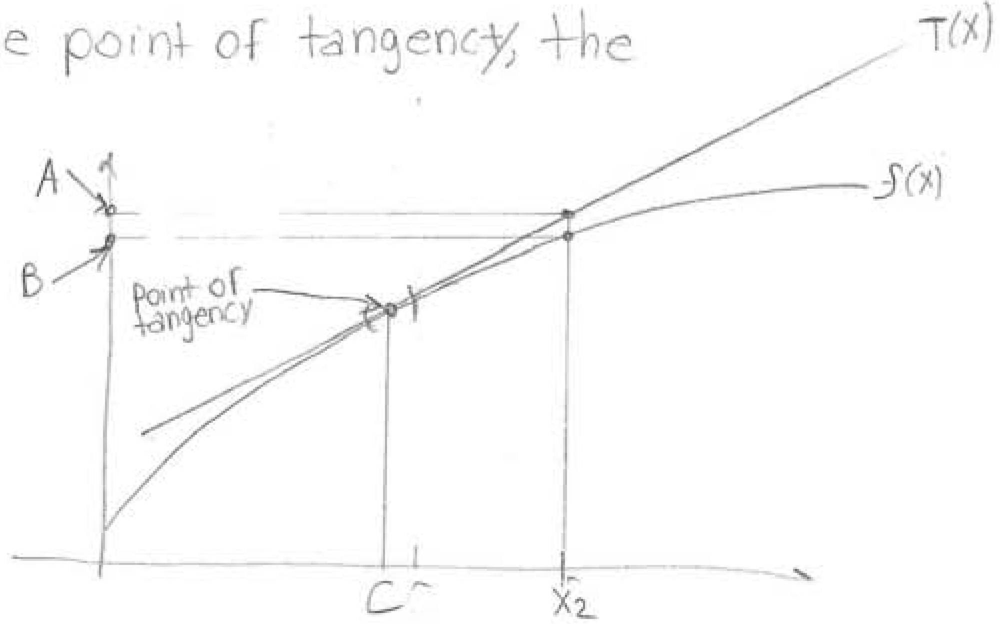
We are using a tangent line to approximate the value of a function.

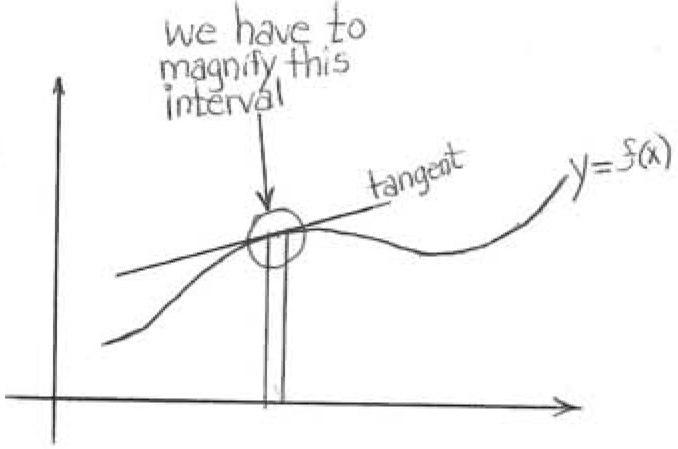
The closer we are to the point of tangency, the more accurate our approximation become



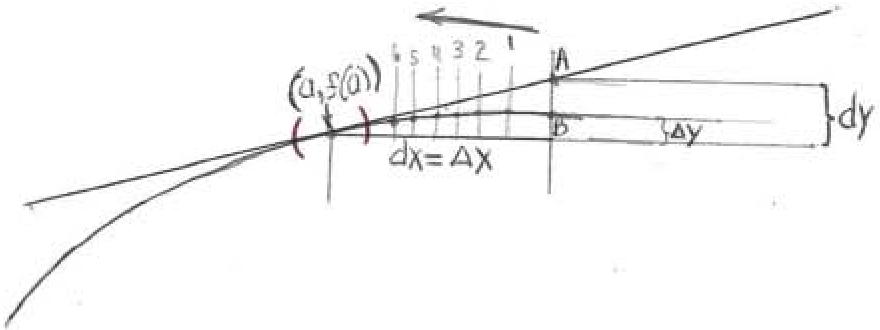
As you see from the graph, the tangent line and f(x) appear to be one and the same within a very small interval. The tangent line appears to overlap f(x) within our very small interval which is depicted on the graph. At the point of tangency, T(x) and f(x) have the same height. T(x) and f(x) appear to have the same height within the very small interval that is shown

As we move further from the point of tangency, the height of $T(x)$ and $f(x)$ are no longer the, same. For example, at x_2, $T(x_2) > f(x_2)$. $T(x) - f(x)$ represents the error at x_2. If we want to approximate $f(x_2)$,

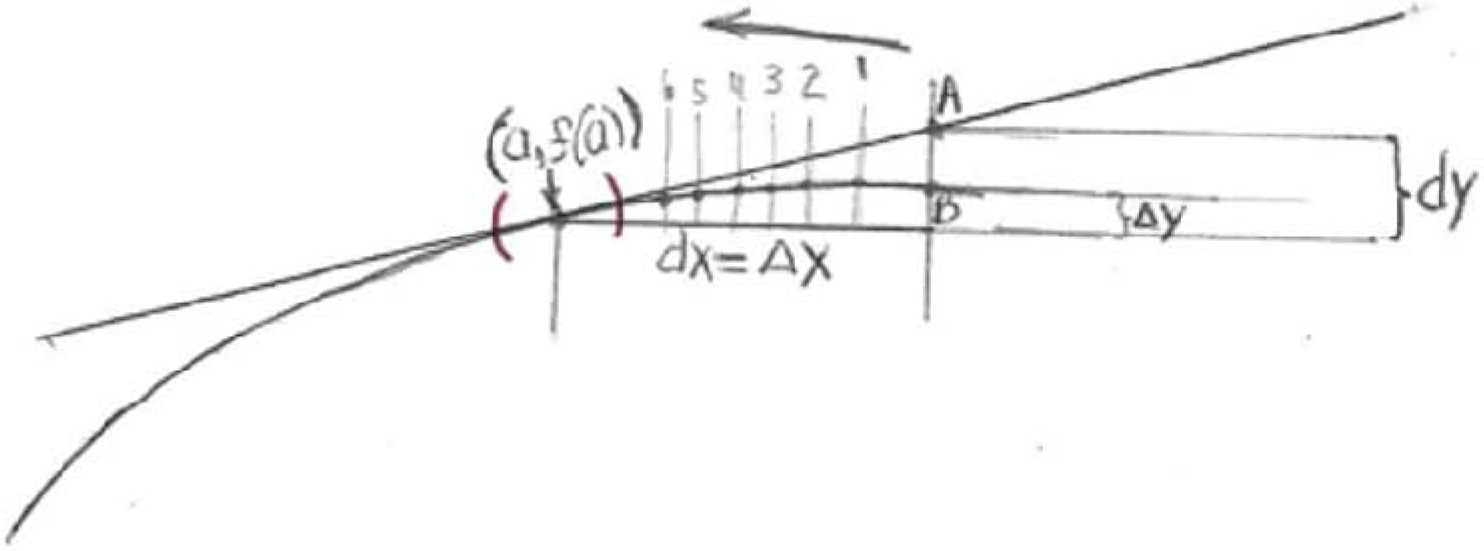
we will have to draw a tangent line at x_2 and encase it in a very small interval where the tangent line and $f(x)$ appear to overlap and become the same line segment. within the extremely small interval, the tangent line approximation becomes extremely close to the actual valve



Magnified Interval (x600)



Magnified Interval (x600)



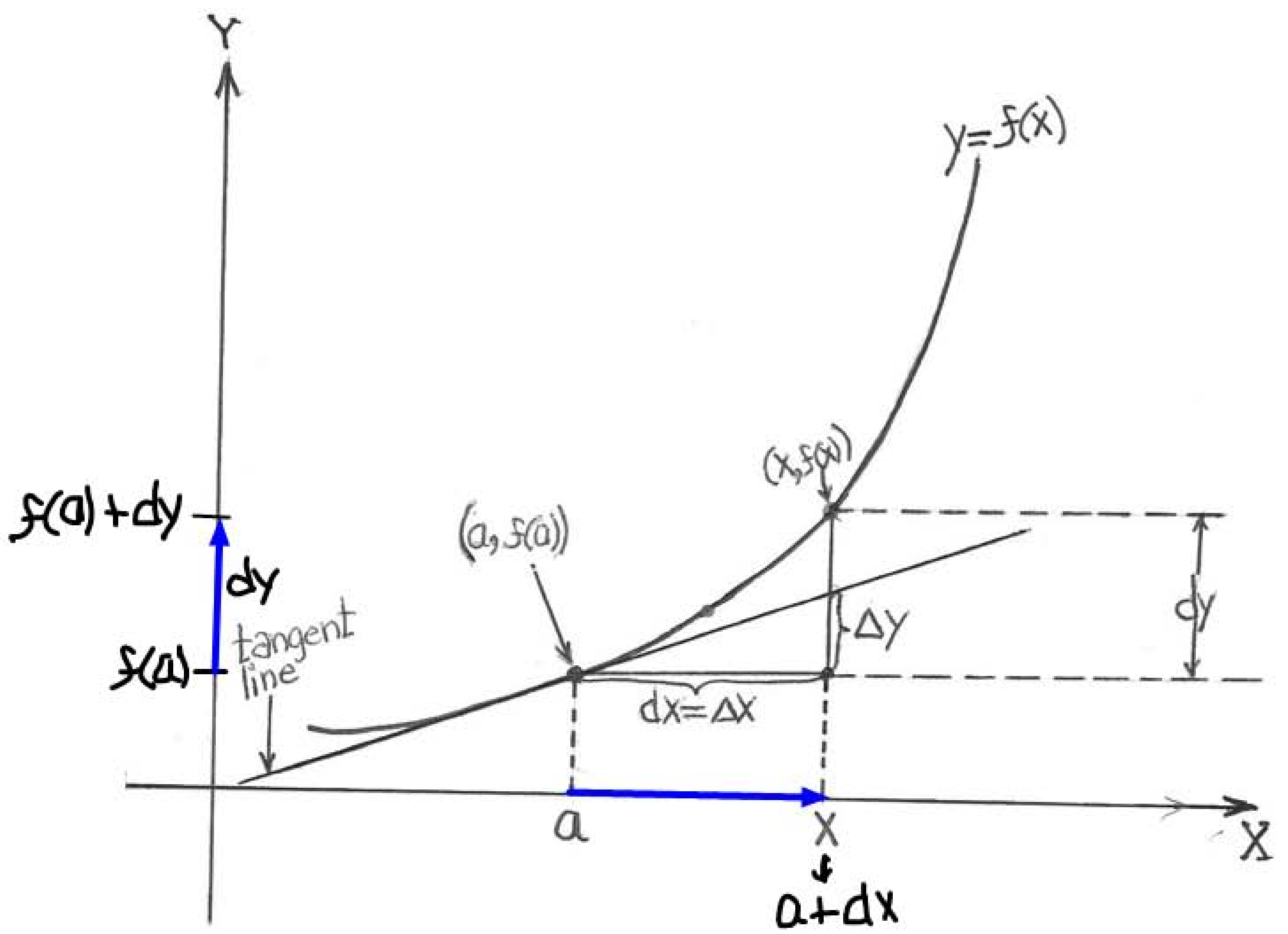
If we envision line segment AB, (with point A remaining on the tangent line and point B remaining on the curve f(x)), moving to the left as it passes through positions 1 to 6, we see that Δy and dy merge at some point beyond position 5; The following happens: $\Delta y = dy$.

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = f'(a)$$

without the $\lim\limits_{\Delta x \to 0}$, we have

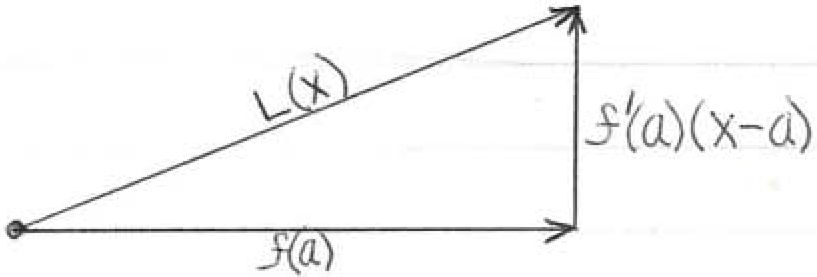
$$\frac{\Delta y}{\Delta x} \approx f'(a)$$

$$\Delta y \approx f'(a)\Delta x$$



Y
y=f(x)
(x,f(x))
f(a)+dy
(a,f(a))
dy
Δy
dy
f(a)
tangent line
dx=Δx
a
X
a+dx
X

If we think in terms of vector addition from physics, $L(x)$ can be thought of as quasi vector addition.



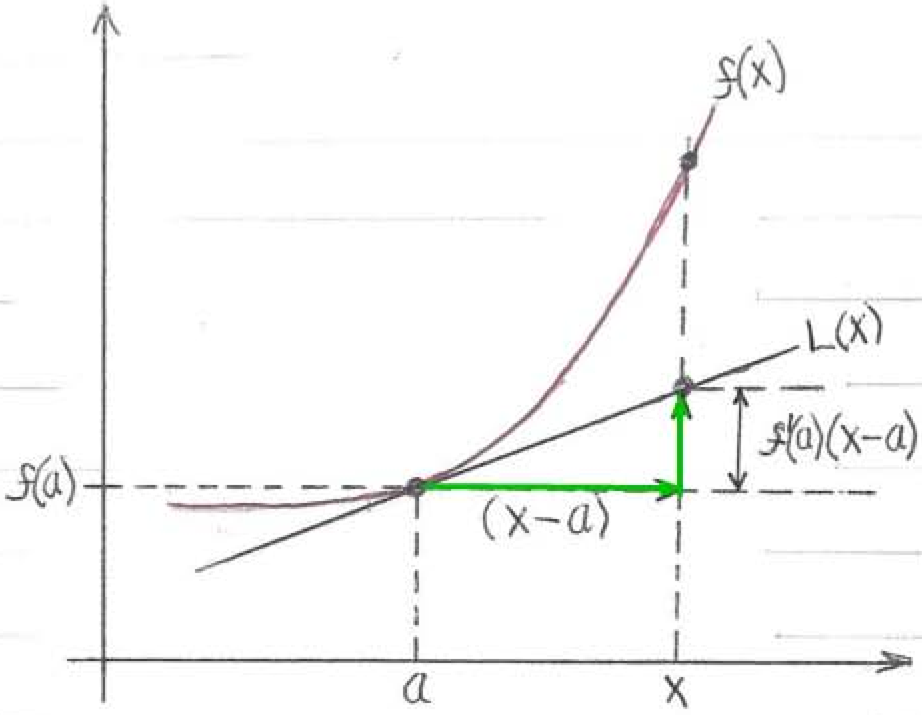
$$L(x) = f(a) + f'(a)(x-a) \qquad (1)$$

The equation (1) is also know as the "Point-Slope Formula."

In equation (1), we sometimes replace "x−a" with Δx.

Equation (1) then becomes

$$L(x) = f(a) + f'(a)\,\Delta x \quad (2)$$



Recap

$$L(x) = f(a) + f'(a)(x-a)$$

or

$$L(x) = f(a) + f'(a)\Delta x$$

$$f(x) \approx L(x) = f(a) + f'(a)(x-a)$$