AB Calculus Related Rates Worksheet 2.6D

1) A spherical balloon is being inflated at a rate of 125π feet cubed per minute. How fast is the radius changing when the radius is 7 ft?

2) A ripple in a pond has a radius that is increasing at a rate of 3cm per minute. How fast is the area of the circular ripple changing when the radius is 20 cm?

3) A 30ft ladder is leaning against a wall. If the top of the ladder is sliding down the wall at a rate of 3 feet per second, how fast is the base of the ladder sliding away from the wall when it is 18ft from the wall?

4) A conical tank is 8ft across and 15ft deep. Water is flowing into the tank at a rate of 12ft cubed per minute. How fast is the depth of the water changing when it is 6ft deep?

5) A balloon is released from the ground tethered by a 130yd string. Its height in the air is going up at a rate of 5 yards per second. How fast is the angle of elevation from the ground changing when the balloon reaches 75yds high? Assume the observer is 40yds away. (Hint: Be careful about being given any extra un-needed information.)

6) A light is at the top of a 18-ft pole. A boy 6 ft tall walks away from the pole at a rate of 3 ft/sec. At what rate is the tip of his shadow moving when he is 24 ft from the pole? At what rate is the length of his shadow increasing?

and 9, 10, 13B, 22, 27-30, 37 from 2.6B

↳ Hint (y, x^2+1) and $(0,0)$ are points + use distance formula

60 pts

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9, 10, 13B, 22
27-30, 37

2.6 D

(9) (x, x^2+1) and $(0,0)$ $\quad \frac{dx}{dt} = 2$ cm/sec

$d = \sqrt{x^2 + (x^2+1-0)^2} \Rightarrow d = \sqrt{x^2 + x^4 + 2x^2 + 1} \Rightarrow d = \sqrt{x^4+3x^2+1}$

$\frac{dD}{dt} = \frac{1}{2}(x^4+3x^2+1)^{-1/2}(4x^3+6x)\frac{dx}{dt}$

$\boxed{\frac{dD}{dt} = \frac{2x^3+3x}{\sqrt{x^4+3x^2+1}}\frac{dx}{dt}}$

at $x=-1$ $\quad \frac{-2-3}{\sqrt{1+3+1}}(2) \quad \frac{-10}{\sqrt{5}} = \boxed{-4.472 \text{ cm/sec}}$

$x=0$ $\quad \frac{0}{1}(2) \quad 0 = \boxed{0 \text{ cm/sec}}$

$x=1$ $\quad \frac{2+3}{\sqrt{1+3+1}}(2) \quad \frac{10}{\sqrt{5}} = \boxed{4.472 \text{ cm/sec}}$

$x=3$ $\quad \frac{54+9}{\sqrt{81+27+1}}(2) \quad \frac{63}{\sqrt{109}}(2) = \boxed{12.069 \text{ cm/sec}}$

(10) $(x, \sin x)$ and $(0,0)$ $\quad \frac{dx}{dt} = 2$ cm/sec

$d = \sqrt{(x-0)^2+(\sin x - 0)^2} \Rightarrow d = \sqrt{x^2+(\sin x)^2}$

$\frac{dD}{dt} = \frac{1}{2}(x^2+\sin^2 x)^{-1/2}(2x + 2\sin x \cos x)$

$\boxed{\frac{dD}{dt} = \frac{x+\sin x \cos x}{(x^2+\sin^2 x)^{1/2}}\frac{dx}{dt}}$

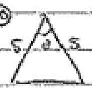
at $x = \frac{\pi}{6}$ $\quad \frac{\pi/6 + (1/2)(\sqrt{3}/2)}{[\pi^2/36 + (1/2)^2]^{1/2}}(2) \Rightarrow 2.643$ cm/sec

$x = \frac{\pi}{4}$ $\quad \Rightarrow 2.433$ cm/sec

$x = \frac{\pi}{3}$ $\quad \Rightarrow 2.179$ cm/sec

$x = \frac{\pi}{2}$ $\quad \Rightarrow 1.687$ cm/sec

Don't show on 13A

(13) (b) 

$A = \frac{1}{2}S^2 \sin\theta$ step 4 $\quad$ step 5: $\frac{dA}{dt} = \frac{1}{2}S^2\cos\theta\frac{d\theta}{dt}$

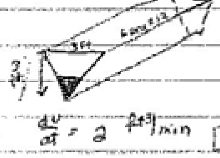
$\left.\begin{array}{l}\frac{d\theta}{dt} = .5 \text{ rad/min} \\ \theta = \frac{\pi}{6} \text{ \& } \frac{\pi}{3}\end{array}\right\}$ step 2 $\quad \frac{dA}{dt} = \frac{1}{2}S^2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$

$\boxed{\frac{dA}{dt} = \frac{\sqrt{3}}{8}S^2 \text{ rad/min}}$

Find $\frac{dA}{dt}$ step 3 $\quad \frac{dA}{dt} = \frac{1}{2}S^2\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

$\boxed{\frac{dA}{dt} = \frac{1}{8}S^2 \text{ rad/min}}$

(c) is $d\theta/dt$ is constant then $\frac{dA}{dt}$ is proportional to $\cos\theta$

(22) 

step 4: $V = BL$

$V = \frac{1}{2}bh \cdot L$

$V = 6bh$

$V = 6h^2$

step 2 $\quad \frac{dV}{dt} = 2$ ft³/min $\quad \boxed{h=1}$

$b = h$ cause $3=3$ $\quad$ step 5 $\frac{dV}{dt} = 12h\frac{dh}{dt}$

step 3 $\quad$ Find $\frac{dh}{dt}$ $\quad$ step 6: $2 = 12(1)\frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = \frac{1}{6} \text{ ft/min}}$

27)

$D = RT$ $\quad 150 = 450T \quad \frac{1}{3}\text{hr} = T$

$D = RT$ $\quad 200 = 600T \quad \frac{1}{3}\text{hr} = T$

$S^2 = P_1^2 + P_2^2$

$2S\frac{dS}{dt} = 2P_1\frac{dP_1}{dt} + 2P_2\frac{dP_2}{dt}$

$\frac{dP_1}{dt} = -450 \qquad \frac{dP_2}{dt} = -600$

$2(250)\frac{ds}{dt} = 2(150)(-450) + 2(200)(-600)$

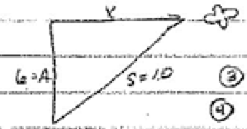
$\boxed{\frac{ds}{dt} = -750\text{mph}}$

$150^2 + 200^2 = \boxed{S = 250}$

⑧ $D = RT \qquad 250 = 750T$

$\boxed{T = \frac{1}{3}}$

28)



$\frac{ds}{dt} = 240$ miles/hr. $\quad$ find $S = 10$

③ $\left[\frac{dx}{dt}\right] \qquad \therefore X = 8$

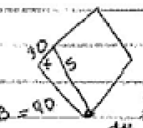
④ $X^2 + A^2 = S^2$

⑤ $X\frac{dx}{dt} + A\frac{dA}{dt} = S\frac{ds}{dt}$

⑥ $8\frac{dx}{dt} + 6(0) = 10(240)$

$\boxed{\frac{dx}{dt} = \frac{2400}{8} = 300\text{mph}}$

29)



getting smaller
-ing

③ Find $\frac{ds}{dt}$

④ $S^2 = X^2 + B^2$

⑤ $2S\frac{ds}{dt} = 2X\frac{dx}{dt} + 2B\frac{dB}{dt}$

$S\frac{ds}{dt} = X\frac{dx}{dt} + B\frac{dB}{dt}$

② $X = 30 \quad \frac{dx}{dt} = -28$

$\therefore S = 94.868$

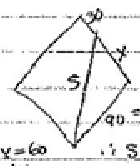
⑥ $(94.868)\frac{ds}{dt} = 30(-28) + 90(0)$

$\frac{ds}{dt} = \frac{-840}{94.868}$

$\boxed{\frac{ds}{dt} = -8.85\,\text{ft/sec}}$

$\boxed{-8.85\,\text{ft/sec}}$

30)



$X = 60$

$\frac{dx}{dt} = 28$

$\therefore S = 108.167$

③ Find $\frac{ds}{dt}$

④ $S^2 = B^2 + X^2$

⑤ $S\frac{ds}{dt} = X\frac{dx}{dt} + B\frac{dB}{dt}$

$(108.167)\frac{ds}{dt} = 60(28) + 90(0)$

$\boxed{\frac{ds}{dt} = 15.532\,\text{ft/sec}}$

31) $pV^{1.3} = k \qquad$ Find $\frac{dp}{dt}$ and $\frac{dV}{dt}$

$\boxed{p(1.3)(V)^{.3}\frac{dV}{dt} + V^{1.3}\frac{dp}{dt} = 0}$

or

$\boxed{1.3pV^{.3}\frac{dV}{dt} = -V^{1.3}\frac{dp}{dt}}$