(#5) Asymptotes

Graph the function

$$y(x)=\frac{x^3+4}{2x^2+x-1}$$

solution

$\Rightarrow$ Y-intercept, $x=0$

$$y(0)=\frac{4}{-1}=-4 \;;\; (0,-4)$$

$\Rightarrow$ x-intercept; $y(x)=0$

$$\frac{x^3+4}{2x^2+x-1}=0$$

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(#5) Asymptotes

$$x^3 + 4 = 0$$

$$x^3 = -4$$

$$x \doteq -1.6 \; ; \; (-1.6, 0)$$

$\Rightarrow$ Vertical Asymptotes

Factor the denominator

#5

Asymptotes

$$2x^2 + x - 1 = 0$$

$$2(-1) = -2$$

$$2x^2 + 2x - x - 1 = 0$$

Middle Term

$$(2x^2 + 2x) + (-x - 1) = 0$$ Group

$$2x(x + 1) - (x + 1) = 0$$ Factor

(#5) Asymptotes

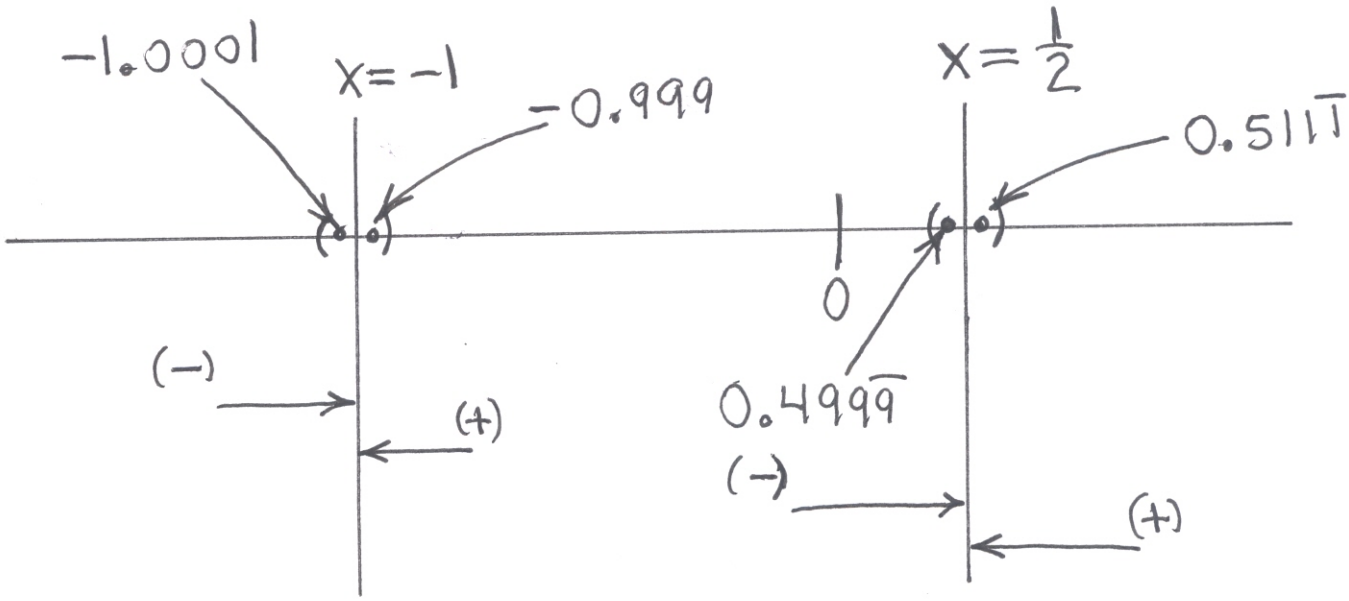
$$(2x-1)(x+1)=0$$

$$x=\frac{1}{2} \quad \& \quad x=-1$$

We must analyze the behavior of $y(x)$ with x-values very, very close to

$$x=\frac{1}{2} \quad \& \quad x=-1$$

(#5) Asymptotes



$$y(x) = \frac{x^3+4}{(2x-1)(x+1)}$$

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#5

Asymptotes

$$\lim_{x \to -1^-} \frac{x^3+4}{(2x-1)(x+1)} \Rightarrow \frac{(+)}{(-)(-)} \Rightarrow (+)$$

$$\lim_{x \to -1^-} y(x) \Rightarrow +\infty$$

$$\lim_{x \to -1^+} \frac{x^3+4}{(2x-1)(x+1)} \Rightarrow \frac{(+)}{(-)(+)} \Rightarrow (-)$$

$$\lim_{x \to -1^+} y(x) \Rightarrow -\infty$$

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(#5)

Asymptotes

$$\lim_{x \to \frac{1}{2}^-} \frac{x^3+4}{(2x-1)(x+1)} \Rightarrow \frac{(+)}{(-)(+)} \Rightarrow (-)$$

$$\lim_{x \to \frac{1}{2}^-} y(x) \Rightarrow -\infty$$

$$\lim_{x \to \frac{1}{2}^+} \frac{x^3+4}{(2x-1)(x+1)} \Rightarrow \frac{(+)}{(+)(+)} \Rightarrow (+)$$

$$\lim_{x \to \frac{1}{2}^+} y(x) \Rightarrow +\infty$$

#5) Asymptotes

⇒ Slant Asymptote

since the power of the leading term in the numerator is one greater than the power of the leading term in the denominator, a slant asymptote exists.

(#5) Asymptotes

$$
\begin{array}{r|l}
 & \frac{1}{2}x - \frac{1}{4} \\
\hline
2x^2+x-1 & x^3 + 0x^2 + 0x + 4 \\
 & -\left(x^3 + \frac{1}{2}x^2 - \frac{1}{2}x\right) \\
\hline
 & -\frac{1}{2}x^2 + \frac{1}{2}x + 4 \\
 & -\left(-\frac{1}{2}x^2 - \frac{1}{4}x + \frac{1}{4}\right) \\
\hline
 & \frac{3}{4}x + \frac{15}{4}
\end{array}
$$

$$y(x) = \left(\frac{1}{2}x - \frac{1}{4}\right) + \frac{\left(\frac{3}{4}x + \frac{15}{4}\right)}{2x^2+x-1}$$

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(#5)

Asymptotes

As $x \to \pm\infty$

$$\frac{\left(\frac{3}{4}x + \frac{15}{4}\right)}{2x^2 + x - 1} \longrightarrow 0$$

The slant asymptote is

$$y = \frac{1}{2}x - \frac{1}{4}$$

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#5

Asymptotes

$$y(x) = \frac{x^3+4}{2x^2+x-1}$$