Precalculus Rational Functions ; Notes

#1) Graph the rational function below.

$$f(x) = \frac{x^2 - x - 2}{1 - x}$$

Solution

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step#1 ⇒ Find the x-intercepts if they exist

Let $f(x) = 0$ because on the X-Axis, $f(x) = 0$

$$\frac{x^2 - x - 2}{1 - x} = 0$$

$$x^2 - x - 2 = (1 - x) \cdot 0$$ cross multiply by $(1-x)$

$$x^2 - x - 2 = 0$$

Precalculus Rational Functions ; Notes

#1)

$$x^2 - x - 2 = 0$$

is easily factorable.

$$x^2 - x - 2 = (x+1)(x-2) = 0$$

$x = -1$ or $x = 2$

The zeros or x-intercepts of $f(x)$ are:

$(-1, 0)$ and $(2, 0)$

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Precalculus, Rational Functions ; Notes

#1) step#2 ⇒ Find the y-intercepts if they exist

Let $x = 0$

$$f(0) = \frac{0^2 - 0 - 2}{1 - 0} = \frac{-2}{1} = -2$$

so the Y-intercept is $(0, -2)$

step#3 ⇒ Vertical Asymptotes

The vertical asymptote is determined by the polynomial or polynomials in the denominator.

NOTE!! ⇒ The denominator may not be equal to zero.

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#1) Let X

$$1 - X = 0$$
$$+X = +X$$
$$1 = X$$

$X = 1$ is the vertical asymptote.

step#4: Slant Asymptote

Since the power of the leading term in the numerator is greater than the power of the leading term in the denominator, a slant asymptote exists.

Precalculus, Rational Functions, Notes

#1) step#4: slant Asymptote

We will use long division and limits to determine the slant asymptote.

$$\begin{array}{r|l} & \quad -x \\ \hline -x+1 & x^2 - x - 2 \\ & \underline{-(x^2 - x)} \\ & \qquad\quad -2 \end{array}$$

so

$$\frac{x^2-x-2}{1-x} = -x + \frac{-2}{1-x}$$

$$x \to \pm\infty \, , \quad \frac{-2}{1-x} \to 0$$

$y = -x$ is the slant asymptote.

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#1)

step#5 ⇒ Determine the behavior of $f(x)$ as its x values get microscopically close to $x=1$.

As the x values approach 1 from the left or from the numbers, we have the following symbolism:

$$x \to 1^-$$

we choose $x = 0.9999$

$$f(x) = \frac{(x+1)(x-2)}{1-x}$$

$$f(0.9999) = \frac{(0.9999+1)(0.9999-2)}{0.9999-1}$$

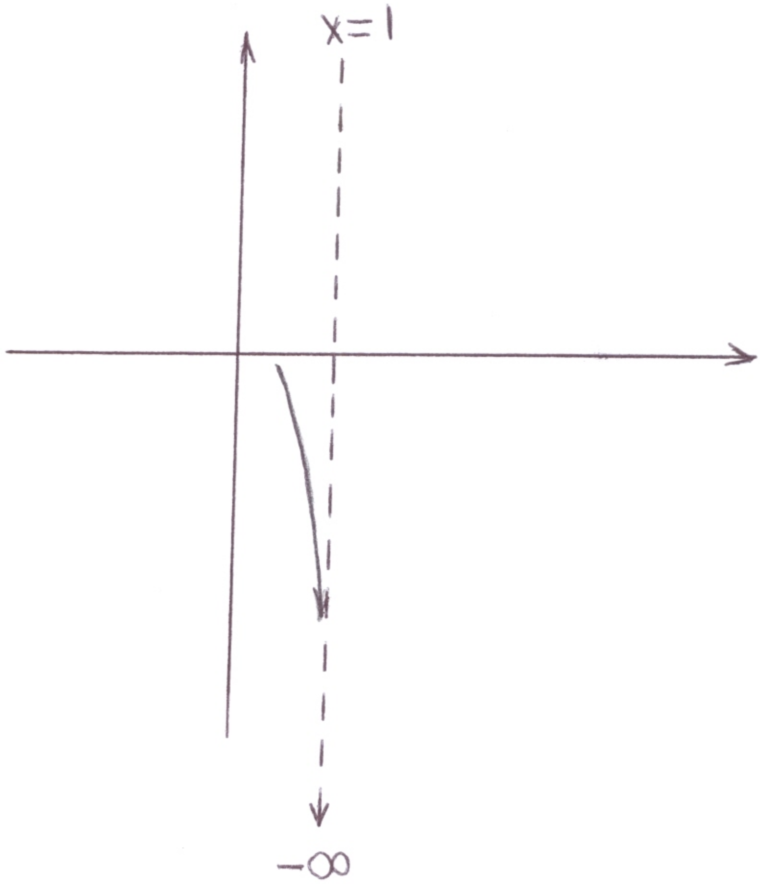
Precalculus, Rational Functions ; Notes

#1)

$$f(0.9999) = \frac{(-)}{(+)} = (-)$$

So,

$f(x) \to -\infty$ as $x \to 1^-$



Precalculus, Rational Functions; Notes

#1)

We will now examine the behavior of $f(x)$ as its x values approach the vertical asymptote $x=1$ from the right or from the positive numbers.

we have the following symbolism:

$$x \to 1^{+}$$

we choose the value $x = 1.00001$

$$f(x) = \frac{(x+1)(x-2)}{1-x}$$

$$f(1.00001) = \frac{(1.00001+1)(1.00001-2)}{(1-1.00001)}$$

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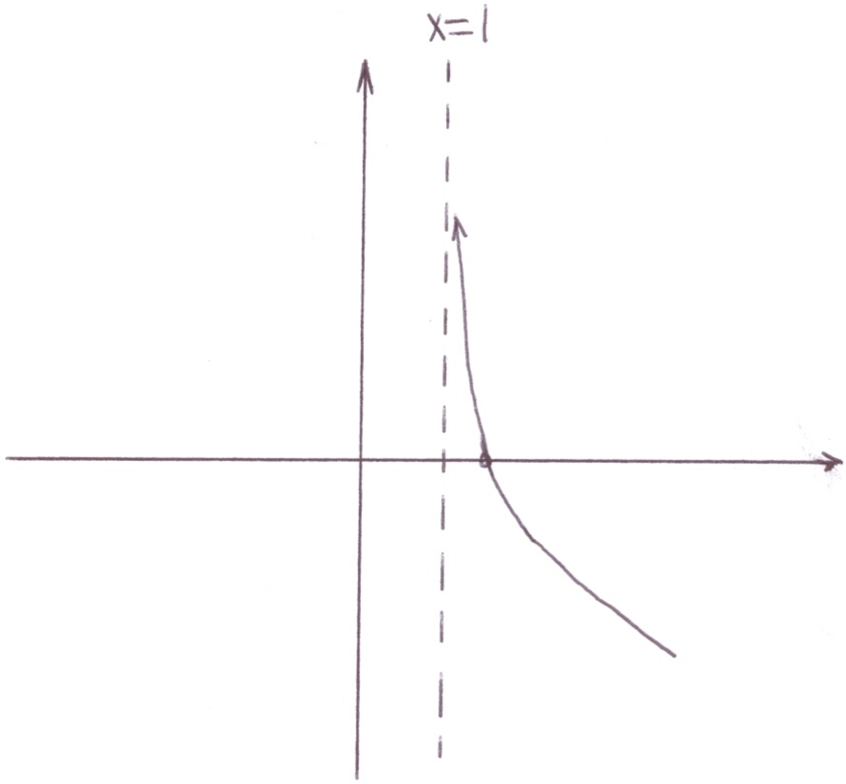
Precalculus, Rational Functions; Notes

#1)

$$f(1.00001) = \frac{(-)}{(-)} = (+)$$

We know the following:

$$f(x) \to +\infty \quad \text{as} \quad x \to 1^{+}$$



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#1) We can now use this information to construct a general graph of this function.